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A new grey decision model-based reference point method for decision makers and criteria's weight, and final ranking

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Abstract

Multi-criteria decision-making (MCDM) challenges are expanding rapidly. Introducing a comprehensive decision model under uncertainty that considers the weight of criteria, the importance of experts, and the ranking of alternatives is essential for solving the MCDM problems. This paper aims to introduce a comprehensive decision model. For this purpose, a developed version of the grey relational analysis (GRA) method by using reference point approach is presented for ranking of alternatives. Moreover, an extension of the best-worst method (BWM), namely G-BWM, is applied for criteria weight determination. Furthermore, the multi-attributive border approximation area comparison (MABAC) method is enhanced by the average ideal concept to specify the weight of experts. The comprehensive model is enriched by employing grey numbers to cope with the uncertainty. To represent the usability of the proposed method, an illustrative example is solved. The outcomes illustrate the reliability of the comprehensive approach, and it can be applied to various MCDM problems.

Keywords: Multi-criteria decision-making, grey relational analysis, reference point method, MABAC, best-worst method, experts' weight

1- Introduction

Choosing the most crucial alternative from a group of alternatives according to the applicable criteria is always one of the most critical issues of reality. In the last decade, some MCDM methods have been extended to solve this problem. Julong (2010) utilized grey target for extending a grey target decision-making and entropy method. Liu et al. (2013) introduced a grey target decision model employing measure functions of uniform effect. Stanujkic et al (2017) solved the best capital investment project problem according to the operational competitiveness rating (OCRA) approach under interval grey sets. Qian et al. (2019) extended an MCDM method according to the regret theory and EDAS method under a grey environment. Wang *et al.* (2020) integrated grey decision-making and fuzzy QFD for solving supply chain problems. Javed *et al.* (2020) expressed a GDM approach according to the grey absolute decision analysis (GADA) under the uncertainty. Ulutaş et al. (2021) selected the best location for a warehouse construction trough a new integrated method under grey sets. Wang et al. (2022) integrated a new MCDM method by using the DEA and grey sets for evaluating the power plants sites.

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Grey relational analysis (GRA) was initially proposed by Julong (1989) that is classified as a known method. The relationships among criteria can be considered by applying the GRA method (Wei et al., 2011; Aydemir and Sahin, 2019; Mahmoudi et al., 2020; Jahangirzade et al., 2021). Maidin et al. (2022) utilized GRA method to select the best material of natural fibre. The GRA method can detect the correlation among the reference and comparable sequences for making the right decisions. Then, the alternatives were ranked by using the computed correlation amounts (Chen, 2019). Furthermore, the GRA method has received considerable attention from researchers because of being easy to understand, easy to use, and does not require mathematical or statistical data (Liu et al., 2019; Ayağ and Samanlioglu, 2020).

The reference point method (RPM) is categorized as a known method that scholars have increasingly utilized in the last decade. The RPM uses Tchebycheff's min-max metric and computes the only distance from the ideal solution (Baležentis and Zeng, 2013; Mi et al., 2020). Dorfeshan et al. (2018) extended the RPM according to the distance from the negative solution. The RPM has significantly been used owing to the being easy to use, defining the positive and negative ideal solutions (PANIS), and simple calculation process (Dorfeshan et al., 2018; Abdi, 2018; Stanujkic et al., 2019; Lin et al., 2020). Kundakcı (2022) expressed a new MCDM method through the system ratio, refrence point, and SWARA method for selecting the energy service companies. To use the GRA method and the RPM benefits, the GRA approach is developed by the RPM to make the right decisions. Also, in the group MCDM problems, the weight of each expert is different from each other because of different skills and varied experiences and views.

To consider the weight of experts in the MCDM procedure, many MCDM methods have been applied recently. Gonçalves et al. (2019) introduced the categorical-based evaluation technique (MACBETH) for evaluating the competitiveness of enterprises. Yue (2012) presented a developed version of the TOPSIS method conforming to the average ideal solution (AIS) to specify the experts' weight. Haghighi et al. (2019) determined the significance of experts conforming to a developed MCDM approach. Mohagheghi et al. (2019) identified the importance of experts according to a new extended TOPSIS approach under interval type-2 fuzzy sets. Dorfeshan and Mousavi (2019) applied a newly developed COPRAS method to the weight of experts' determination. Yang et al. (2020) expanded a new MCDM technique to DMs' weight determination procedure in MCGDM problems. Salimian et al. (2022) evaluated the infrastructure projects and determined the weight of experts by means of the WASPAS method. All of the previous extended methods are based on the AIS; one of the MCDM methods that initially is conforming to AIS for ranking of alternatives is the multi-attributive border approximation area comparison (MABAC) approach.

One of the newly presented MCDM methods is MABAC that is proposed by (Pamučar and Ćirović, 2015). The MABAC approach due to the stability in solution, easy to use, and potential quantity of merits and demerits that to be defined so that the outcome can be tremendous (Dorfeshan et al., 2020). Liang et al. (2019) evaluated the risks of rockburst using a new MABAC approach according to the fuzzy environment. Wei et al. (2019) selected the best supplier for medical consumption products based on a new extended MABAC method. Ghadikolaei et al. (2022) assessed suppliers from the point of green view through a combination of MABAC and ANP methods. In this research, because of the nature of the MABAC approach, it is developed and used for the experts' weight determination process.

In a decision-making process, the criteria's weight determination is very crucial. Many weighting approaches have been extended in the last two decades. For instance, Akbarzadeh et al. (2019) applied the DEMATEL and ANP techniques to specify the weight of criteria in a supply chain practice selection procedure. Tavakoli et al. (2011) utilized the AHP method to determine the impotance of criteria in the plant location selection process. Kakha et al. (2019) used the DEMATEL method for criteria weighting determination in sustainable mininng development problems. One of the newest MCDM techniques is the best-worst method (BWM) that is offered initially by (Rezaei, 2015). Then, Rezaei (2016) improved the old version of BWM by converting the non-linear model to a linear model. The BWM method has some advantages in comparison with the AHP method. BWM method requires just $2n-3$ comparisons for pairwise comparing of n criteria, and it provides more consistent comparisons. In this paper, to achieve a reliable weight for criteria, the BWM method is extended and applied.

Moreover, for addressing the proposed approach's uncertainty, grey numbers are applied. The grey numbers are used in this paper because 1) the grey numbers in comparisons of other uncertainty approaches require fewer data and 2) the ability of grey numbers to model the real-world's uncertainty of MCDM problems (Stanujkic et al., 2017). Oztaysi (2014) used the TOPSIS and AHP methods for the ranking of information technology under the grey environment. According to the RPM and new MABAC technique for weighting of experts, the proposed GRA is developed under the grey numbers to cope with the vagueness and uncertainty.

Based on the above statements, the GRA method is developed by the RPM to use both merits of these two methods. Furthermore, new experts' weight determination based on the MABAC method is expanded. Moreover, the criteria weight determination process is done based on the extended BWM method under the grey environment. Finally, all extended methods for ranking and weighting are developed utilizing grey numbers. The paper's novelties are explained below:

- On the one hand, the GRA method can detect the correlation among the reference and comparable sequence for making the right decisions; on the other hand, the RPM has significantly been used owing to the being easy to use, considering the PANIS, and simple calculation process. To use the GRA and RPM advantages, the GRA technique is developed by RPM.
- According to Yue's (2011) concept for determining the weight of experts based on the AIS, the MABAC technique is developed according to the AIS and border approximation area to determine the importance of experts in GDM. The MABAC method is extended for weight determination.
- The weight of the criteria is determined by using the BWM method under the grey environment. The G-BWM method compares the *n* criteria with the *2n-3* pairwise comparisons. Compared to the AHP method, the proposed method takes less time.
- The comprehensive decision model contains the ranking of alternatives, and weighing criteria and experts are extended under the grey uncertainty tool to cope with the uncertainty of practical problems.

This manuscript is formed from 5 sections: Section 2 explains the basic knowledge. Section 3 proposes a new MCDM method. Section 4 presents an illustrative example for demonstrating the calculation process of the defined method. Chapter 5 concludes the crucial remarks.

2- Preliminary knowledge of grey system

The interval grey numbers concept was initially introduced by (Julong, 1989). The grey numbers are more potent than deterministic numbers for modeling the uncertainty of real-world positions. Many well-known MCDM methods have been extended under the grey environment in the last decade (Turskis and Zavadskas, 2010; Turskis et al., 2016). The essential operation of grey numbers is defined as below (Liu et al., 2017):

$$\otimes \beta_1 + \otimes \beta_2 = [\underline{\beta}_1 + \underline{\beta}_2, \bar{\beta}_1 + \bar{\beta}_2]$$

$$\otimes \beta_1 - \otimes \beta_2 = [\underline{\beta}_1 - \bar{\beta}_2, \bar{\beta}_1 - \underline{\beta}_2]$$

$$\otimes \beta_1 \times \otimes \beta_2 = [\underline{\beta}_1 \underline{\beta}_2, \bar{\beta}_1 \bar{\beta}_2]$$

$$\otimes \beta_1 \div \otimes \beta_2 = [\frac{\underline{\beta}_1}{\bar{\beta}_2}, \frac{\bar{\beta}_1}{\underline{\beta}_2}]$$

$$\mu \otimes \beta_1 = \mu \otimes \left[\underline{\beta}_1, \bar{\beta}_1\right] = \left[\mu \underline{\beta}_1, \mu \bar{\beta}_1\right]$$

Note that μ is a positive actual number.

3- A new MCDM model

In this part, the GRA method is extended according to the RPM and Tchebycheff's min-max metric. Furthermore, experts' weight determination is very crucial for making the right decision in the GDM procedure. Hence, a new MABAC method is developed for experts' weight determination. Then, the extended ranking and weighting method are expanded under the grey environment. Also, to achieve a comprehensive decision model, a new version of BWM (G-BWM) is proposed for criteria weight determination.

Step 1. A group of experts is constructed, and the rankings of alternatives based on the essential criteria are gathered from the experts. The equivalents of linguistic variables for grades are tabulated in table 1.

Table 1. Linguistic terms for ratings of alternatives and their corresponding grey numbers

Linguistic variables	**Equivalent grey number**
Very Poor (VP)	(0, 1)
Poor (P)	(1, 3)
Medium Poor (MP)	(3, 5)
Fair (F)	(5, 7)
Medium Good (MG)	(7, 8)
Good (G)	(8, 9)
Very Good (VG)	(9, 10)

Step 2. The h-*th* matrix is formed according to the H experts' views. Equation (1) illustrates the decision matrices.

$$[H_{q\,s}^{\gamma}]_{Q\times S} = \begin{bmatrix} (\underline{H}_{11}^{\gamma}, \overline{H}_{11}^{\gamma}) & \cdots & (\underline{H}_{1\,s}^{\gamma}, \overline{H}_{1\,s}^{\gamma}) \\ \vdots & \ddots & \vdots \\ (\underline{H}_{q\,1}^{\gamma}, \overline{H}_{q\,1}^{\gamma}) & \cdots & (\underline{H}_{q\,s}^{\gamma}, \overline{H}_{q\,s}^{\gamma}) \end{bmatrix} \tag{1}$$

Where $1 \leq q \leq Q$ depicts the amounts of alternatives, $1 \leq s \leq S$ represents the amounts of essential factors, $1 \leq \gamma \leq \beta$ illustrates the number of experts.

Step 3. The experts' weights are determined pursuant to the new MABAC technique. The following sub-steps are used for the weight determination process.

Step 3-1. The border approximation area (average ideal solution) matrix is specified as follows:

$$[H_{q\,s}^{*}]_{Q\times S} = \begin{bmatrix} (\frac{\sum_{\gamma=1}^{\beta}\underline{H}_{11}^{\gamma}}{\beta}, \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{11}^{\gamma}}{\beta}) & \cdots & (\frac{\sum_{\gamma=1}^{\beta}\underline{H}_{1\,s}^{\gamma}}{\beta}, \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{1\,s}^{\gamma}}{\beta}) \\ \vdots & \ddots & \vdots \\ (\frac{\sum_{\gamma=1}^{\beta}\underline{H}_{q1}^{\gamma}}{\beta}, \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{q1}^{\gamma}}{\beta}) & \cdots & (\frac{\sum_{\gamma=1}^{\beta}\underline{H}_{q\,s}^{\gamma}}{\beta}, \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{q\,s}^{\gamma}}{\beta}) \end{bmatrix} \tag{2}$$

Step 3-2. The distance of each matrix from the AIS matrix is specified by:

$$[\theta_{q\,s}^{\gamma}]_{Q\times S} = \begin{bmatrix} (\underline{H}_{11}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{11}^{\gamma}}{\beta}, & \cdots & (\underline{H}_{1\,s}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{1\,s}^{\gamma}}{\beta}, \\ \overline{H}_{11}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\underline{H}_{11}^{\gamma}}{\beta}) & & \overline{H}_{1\,s}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\underline{H}_{1\,s}^{\gamma}}{\beta}) \\ \vdots & \ddots & \vdots \\ (\underline{H}_{q\,1}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{q1}^{\gamma}}{\beta}, & \cdots & (\underline{H}_{q\,s}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{q\,s}^{\gamma}}{\beta}, \\ \overline{H}_{q\,1}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\underline{H}_{q1}^{\gamma}}{\beta}) & & \overline{H}_{q\,s}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\underline{H}_{q\,s}^{\gamma}}{\beta}) \end{bmatrix} \tag{3}$$

Then, the upper and lower limit of each element is integrated below:

$$\frac{\underline{H}_{q\,s}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\overline{H}_{q\,s}^{\gamma}}{\beta} + \overline{H}_{q\,s}^{\gamma} - \frac{\sum_{\gamma=1}^{\beta}\underline{H}_{q\,s}^{\gamma}}{\beta}}{2} \tag{4}$$

Step 3-3. The final value of the weight of experts is obtained by equation (5).

$$\varphi^{\gamma} = \left|\sum_{s=1}^{S}\sum_{q=1}^{Q}\left[\theta_{q\,s}^{\gamma}\right]_{Q\times S}\right| \qquad \forall\gamma = 1,\ldots,\beta \tag{5}$$

Step 3-4. The weight of each expert is determined through equation (6):

$$\xi^{\gamma} = \frac{\varphi^{\gamma}}{\sum_{\gamma=1}^{\beta}\varphi^{\gamma}} \qquad \forall\gamma = 1,\ldots,\beta \tag{6}$$

Step 4. All matrices are aggregated employing the experts' weight by:

$$\left[\vartheta_{q\,s}\right]_{Q\times S} = \frac{\sum_{\gamma=1}^{\beta}\xi^{\gamma}\times\begin{bmatrix}(\underline{H}_{11}^{\gamma},\overline{H}_{11}^{\gamma}) & \cdots & (\underline{H}_{1\,s}^{\gamma},\overline{H}_{1\,s}^{\gamma})\\ \vdots & \ddots & \vdots\\ (\underline{H}_{q\,1}^{\gamma},\overline{H}_{q\,1}^{\gamma}) & \cdots & (\underline{H}_{q\,s}^{\gamma},\overline{H}_{q\,s}^{\gamma})\end{bmatrix}}{\sum_{\gamma=1}^{\beta}\xi^{\gamma}} \tag{7}$$

Step 5. The aggregated matrices are normalized through equation (8).

$$\left[H_{q\,s}\right]_{Q\times S} = \begin{bmatrix}(\underline{H}_{11},\overline{H}_{11}) & \cdots & (\underline{H}_{1\,s},\overline{H}_{1\,s})\\ \vdots & \ddots & \vdots\\ (\underline{H}_{q\,1},\overline{H}_{q\,1}) & \cdots & (\underline{H}_{q\,s},\overline{H}_{q\,s})\end{bmatrix} \tag{8}$$

where,

$$\begin{aligned}(\underline{H}_{q\,s},\overline{H}_{q\,s}) &= \left(\frac{\underline{\vartheta}_{q\,s}-\bar{\vartheta}_{q\,s}^{-}}{\bar{\vartheta}_{q\,s}^{+}-\underline{\vartheta}_{q\,s}^{-}},\frac{\bar{\vartheta}_{q\,s}-\bar{\vartheta}_{q\,s}^{-}}{\bar{\vartheta}_{q\,s}^{+}-\underline{\vartheta}_{q\,s}^{-}}\right) for\ \ benefit\ criteria\\ (\underline{H}_{q\,s},\overline{H}_{q\,s}) &= \left(\frac{\bar{\vartheta}_{q\,s}^{+}-\bar{\vartheta}_{q\,s}}{\bar{\vartheta}_{q\,s}^{+}-\underline{\vartheta}_{q\,s}^{-}},\frac{\bar{\vartheta}_{q\,s}^{+}-\underline{\vartheta}_{q\,s}}{\bar{\vartheta}_{q\,s}^{+}-\underline{\vartheta}_{q\,s}^{-}}\right) for\ \ cost\ criteria\end{aligned} \tag{9}$$

Notably $\bar{\vartheta}_{q\,s}^{+} = \max_{1\le q\le Q}\{\bar{\vartheta}_{q\,s}\}$ and $\underline{\vartheta}_{q\,s}^{-} = \max_{1\le q\le Q}\{\underline{\vartheta}_{q\,s}\}$.

Step 6. The criteria's weight is specified according to the BWM method. The proposed BWM method is extended under a grey environment as G-BWM.
Step 6-1. The worst and best criterion is determined pursuant to the opinions of DMs.
Step 6-2. The precedence of the most desirable factor over the other factors is expressed by employing the linguistic variables illustrated in table 2.

Table 2. Grey equivalent of linguistic terms

Linguistic terms	Grey numbers
Equal importance (E)	[1,1]
Moderate importance (M)	[2,3]
Strong importance (S)	[4,5]
Very strong importance (VS)	[6,7]
Extreme importance (EI)	[8,9]

The best's vector (BV) over other criteria is defined by:

$$\nabla_{BV} = (\nabla_{BV,1},\nabla_{BV,2},\ldots,\nabla_{BV,S}) \tag{10}$$

Notably, each member of BV is a grey number.

Step 6-3. The precedence of other factors over the least desirable factor is explained through the DMs' judgment employing Table II. The other factors' vector (WV) over the worst criterion is defined by:

$$\nabla_{WV} = (\nabla_{WV,1}, \nabla_{WV,2}, \dots, \nabla_{WV,S}) \tag{11}$$

Step 6-4. Two linear models are defined for obtaining the upper and lower weight of each criterion as follows:

$$\begin{aligned}
&\min \underline{\ell} \\
&\left|\Phi^1{}_{BV} - \underline{\nabla}_{BV,s} * \Phi^1{}_s\right| \le \underline{\ell} \quad \forall s = 1, \dots, S \\
&\left|\Phi^1{}_s - \underline{\nabla}_{WV,s} * \Phi^1{}_{WV}\right| \le \underline{\ell} \quad \forall s = 1, \dots, S \\
&\sum_{s=1}^{S} \Phi^1{}_s = 1
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
&\min \overline{\ell} \\
&\left|\Phi^2{}_{BV} - \overline{\nabla}_{BV,s} * \Phi^2{}_s\right| \le \underline{\ell} \quad \forall s = 1, \dots, S \\
&\left|\Phi^2{}_s - \overline{\nabla}_{WV,s} * \Phi^2{}_{WV}\right| \le \underline{\ell} \quad \forall s = 1, \dots, S \\
&\sum_{s=1}^{S} \Phi^2{}_s = 1
\end{aligned} \tag{13}$$

Note that, $\underline{\ell}, \overline{\ell}$ represent the consistency degrees. The lower the value, is better. The final derived weight of criteria from models are $(\Phi^*{}_1, \Phi^*{}_2, \dots, \Phi^*{}_S)$. Moreover, each element of the weight vector is defined as follows:

$$\Phi^*{}_s = \begin{cases} (\underline{\Phi}^*{}_s, \overline{\Phi}^*{}_s) = (\Phi^{1*}{}_s, \Phi^{2*}{}_s) & if \quad \Phi^{1*}{}_s \le \Phi^{2*}{}_s \\ (\underline{\Phi}^*{}_s, \overline{\Phi}^*{}_s) = (\Phi^{2*}{}_s, \Phi^{1*}{}_s) & if \quad \Phi^{1*}{}_s \ge \Phi^{2*}{}_s \end{cases} \tag{14}$$

Step 7. The normalized matrices are multiplied in the final weight of criteria by using the following:

$$\begin{aligned}
&[U_{q\,s}]_{Q\times S} = \Phi^*{}_s \times [H_{q\,s}]_{Q\times S} = \\
&\begin{bmatrix} (\underline{\Phi}^*{}_1 \times \underline{H}_{11}, \overline{\Phi}^*{}_1 \times \overline{H}_{11}) & \cdots & (\underline{\Phi}^*{}_s \times \underline{H}_{1\,s}, \overline{\Phi}^*{}_s \times \overline{H}_{1\,s}) \\ \vdots & \ddots & \vdots \\ (\underline{\Phi}^*{}_1 \times \underline{H}_{q\,1}, \overline{\Phi}^*{}_1 \times \overline{H}_{q\,1}) & \cdots & (\underline{\Phi}^*{}_s \times \underline{H}_{q\,s}, \overline{\Phi}^*{}_s \times \overline{H}_{q\,s}) \end{bmatrix}
\end{aligned} \tag{15}$$

Step 8. The PANIS are specified as follows:

$$\begin{aligned}
(\underline{U}^+_{q\,s}, \overline{U}^+_{q\,s}) &= \begin{bmatrix} (\max\limits_{1\le q\le Q} \underline{U}_{q\,1}, \max\limits_{1\le q\le Q} \overline{U}_{q\,1}), \\ (\max\limits_{1\le q\le Q} \underline{U}_{q\,2}, \max\limits_{1\le q\le Q} \overline{U}_{q\,2}), \dots, \\ (\max\limits_{1\le q\le Q} \underline{U}_{q\,S}, \max\limits_{1\le q\le Q} \overline{U}_{q\,S}) \end{bmatrix} \\
(\underline{U}^-_{q\,s}, \overline{U}^-_{q\,s}) &= \begin{bmatrix} (\min\limits_{1\le q\le Q} \underline{U}_{q\,1}, \min\limits_{1\le q\le Q} \overline{U}_{q\,1}), \\ (\min\limits_{1\le q\le Q} \underline{U}_{q\,2}, \min\limits_{1\le q\le Q} \overline{U}_{q\,2}), \dots, \\ (\min\limits_{1\le q\le Q} \underline{U}_{q\,S}, \min\limits_{1\le q\le Q} \overline{U}_{q\,S}) \end{bmatrix}
\end{aligned} \tag{16}$$

Step 9. The computed distance from PANIS is obtained as below:

$$[\pi^+_{q\,s}]_{Q\times S} = \begin{bmatrix} \pi^+{}_{11} & \cdots & \pi^+{}_{1\,s} \\ \vdots & \ddots & \vdots \\ \pi^+{}_{q\,1} & \cdots & \pi^+{}_{q\,s} \end{bmatrix} \tag{17}$$

$$\left[\pi^-_{q\,s}\right]_{Q\times S} = \begin{bmatrix} \pi^-{}_{11} & \cdots & \pi^-{}_{1\,s} \\ \vdots & \ddots & \vdots \\ \pi^-{}_{q\,1} & \cdots & \pi^-{}_{q\,s} \end{bmatrix} \tag{18}$$

where $\pi^+{}_{q\,s} = \sqrt{(\underline{U}_{q\,s} - \max_{1\le q\le Q} \underline{U}_{q\,s})^2 + (\overline{U}_{q\,s} - \max_{1\le q\le Q} \overline{U}_{q\,s})^2}$,

$\pi^-{}_{q\,s} = \sqrt{(\underline{U}_{q\,s} - \min_{1\le q\le Q} \underline{U}_{q\,s})^2 + (\overline{U}_{q\,s} - \min_{1\le q\le Q} \overline{U}_{q\,s})^2}$.

Step 10. The negative and positive grey relational coefficient matrix $\left[\rho_{q\,s}\right]_{Q\times S}$is calculated as follows:

$$\left[\rho^+_{q\,s}\right]_{Q\times S} = \begin{bmatrix} \rho^+{}_{11} & \cdots & \rho^+{}_{1\,s} \\ \vdots & \ddots & \vdots \\ \rho^+{}_{q\,1} & \cdots & \rho^+{}_{q\,s} \end{bmatrix} \tag{19}$$

$$\left[\rho^-_{q\,s}\right]_{Q\times S} = \begin{bmatrix} \rho^-{}_{11} & \cdots & \rho^-{}_{1\,s} \\ \vdots & \ddots & \vdots \\ \rho^-{}_{q\,1} & \cdots & \rho^-{}_{q\,s} \end{bmatrix} \tag{20}$$

where $\rho^+{}_{q\,s} = \frac{\min_q \min_s \pi^+_{q\,s} + \tau \min_q \min_s \pi^+_{q\,s}}{\pi^+_{q\,s} + \tau \min_q \min_s \pi^+_{q\,s}}$, $\rho^-{}_{q\,s} = \frac{\min_q \min_s \pi^-_{q\,s} + \tau \min_q \min_s \pi^-_{q\,s}}{\pi^-_{q\,s} + \tau \min_q \min_s \pi^-_{q\,s}}$.

Step 11. In the classic RPM, Tchebycheff's min-max metric is used. In this paper, because of the nature of data after applying the grey relational coefficient, the max-min Tchebycheff metric is implemented as follows:

$$\max_q \left(\left[\frac{\min_s \rho^+_{q\,s} + \min_s (1 - \rho^-_{q\,s})}{2}\right]\right) \ \forall q = 1,\ldots,Q \\ and\ s = 1,\ldots,S \tag{21}$$

Note that the higher values get the more top ranks.

4- Illustrative example

An example from literature (Mousavi et al., 2014) is solved for reflecting the calculation process and strengths of PM. The best conveyor of material handling equipment must be selected in the textile manufacturing company. Four conveyors and six critical criteria are defined for this application. The applicable criteria are defined as follows: 1) Fixed cost 2) Variables cost 3) Speed of conveyor 4) Item width 5) Item weight, and 6) Flexibility.

Step A. The data are gathered from a team of experts with three members and converted them based on table I. The initial data are tabulated in tables 3 and 4.

Table 3. Collected data on ratings of conveyors based on the efficient criteria from experts

Criteria	Alternatives	Expert$_1$	Expert$_2$	Expert$_3$
C_1	A_1	P	VP	P
	A_2	F	F	F
	A_3	F	F	F
	A_4	P	P	P
C_2	A_1	P	P	MP
	A_2	P	MP	MP
	A_3	P	P	MP
	A_4	F	F	MG
C_3	A_1	VG	G	G
	A_2	MG	MG	G
	A_3	MG	G	G
	A_4	MP	MP	F
C_4	A_1	15	15	15
	A_2	20	20	20
	A_3	25	25	25
	A_4	35	35	35
C_5	A_1	10	10	10
	A_2	8	8	8
	A_3	20	20	20
	A_4	25	25	25
C_6	A_1	F	F	F
	A_2	MG	G	G
	A_3	G	G	G
	A_4	MG	MG	F

Table 4. Collected data on the significance of criteria from experts

DMs	C_1	C_2	C_3	C_4	C_5	C_6
DM_1	MH	M	M	L	ML	M
DM_2	M	MH	MH	L	M	M
DM_3	H	M	M	ML	ML	M

Step B. Three decision matrices are formed through equation (1).
Step C. The weights of experts are computed through a development of the MABAC method pursuant to equations. (2-6). The final weight of experts is tabulated in table 5.
Step D. Pursuant to the weight of experts, the integrated matrix is computed through equation (7).
Step E. The aggregated decision matrix is normalized by equations (8) and (9).
Step F. Determination of the weight of criteria is done in this step.
Step F-1. From the importance of view, the least and most desirable factors are defined. The least and most desirable elements are depicted in table 6.
Step F-2. The best's vector (BV) over other criteria is defined by equation (10). The BV is displayed in table 6.
Step F-3. The other criteria's vector (WV) over the worst criterion is defined by equation (11). The WV is displayed in table 6.
Step F-4. Two linear models are defined to obtain the upper and lower weight of each criterion employing equations (12)-(14). The final importance of criteria is represented in table 7.

Table 5. The final weight of experts

Experts	Final weights
$Expert_1$	0.37
$Expert_2$	0.43
$Expert_3$	0.2

Table 6. Best and worst criteria and their vector

	C_1	C_2	C_3	C_4	C_5	C_6
Best criterion: C_1	E	M	M	VS	S	S
Worst criterion: C_4	VS	S	S	E	M	M

Table 7. Final weight of criteria

Criteria	Final weight
C_1	[0.36,0.41]
C_2	[0.166,0.194]
C_3	[0.166,0.194]
C_4	[0.05,0.055]
C_5	[0.097,0.1]
C_6	[0.097,0.1]

Step G. The normalized decision matrix is multiplied in the final weight of criteria utilizing equation (15).
Step H. The PANIS is computed through equation (16).
Step I. The matrix of distanced from PANIS is computed through equations (17) and (18).
Step J. The negative and positive grey relational coefficient matrices are obtained by equations (19) and (20).
Step K. The final values and rankings of alternatives are computed by applying the max-min Tchebycheff metric employing equation (21). The final results are tabulated in table 8.

4-1- Comparative analysis

In this part, the outcomes of PM are compared with the outcomes of the TOPSIS method. The results are demonstrated in table 9. The results of the TOPSIS method have confirmed the results of PM. This authentication demonstrates the validity of PM.

Table 8. The final values and rankings of alternatives

Alternatives	Final values	Final rankings
Conveyor 1	0.662407	1
Conveyor 2	0.409343	2
Conveyor 3	0.366921	3
Conveyor 4	0.166667	4

Table 9. Results of comparative analysis

Alternatives	Final values	Final rankings	Final results of the TOPSIS method	Final rankings
Conveyor 1	0.6624	1	0.97865	1
Conveyor 2	0.4093	2	0.69081	2
Conveyor 3	0.3669	3	0.62639	3
Conveyor 4	0.1666	4	0	4

4-2- Different degree analysis

To display the supremacy of the PM in comparisons with the TOPSIS method, a different degree is computed. When there are sorted values of alternative in descending order, for example, the S and Q, the different degree is calculated by:

$$DD = \frac{S\ value - Q\ value}{Q\ value} \quad , S\ value \geq Q\ value$$

If the two methods' ranking is precisely the same, the different degree values will be decisive. A method with a higher value is better than others (Wu et al., 2018). The various degree values of each method are computed and tabulated in table 10. As can be seen, the results ascertain the superiority of the PM over the TOPSIS method. Unlike TOPSIS, the proposed method in this paper consists of three parts. In one part, the weight of the experts is determined. In other parts, the weight of the criteria and finally the final ranking of the alternatives are specified, respectively. Also, the GRA method can detect the correlation among the reference and comparable sequences for making the right decisions. Furthermore, the alternatives are ranked by using the computed correlation amounts. While TOPSIS does not consider the above cases.

A different degree indicates the dispersion value of alternatives. The greater difference between the final values of the alternatives leads to the better and more stable results because the rankings will not change easily with a slight change in the values of the decision matrix.

Table 10. Different degree values of each method

Alternatives	Final values	Different degree values	Final results of the TOPSIS method	Different degree values
Conveyor 1	0.6624	0.61822	0.97865	0.416681
Conveyor 2	0.4093	0.115616	0.69081	0.102829
Conveyor 3	0.3669	1.201526	0.62639	0
Conveyor 4	0.1666		0	

5- Conclusion

In this manuscript, a novel method has been developed. The GRA technique has been extended by using the RPM for achieving stable and durable results. Moreover, instead of Tchebycheff's min-max metric, Tchebycheff's max-min metric has been applied. To increase the reliability of making the right decisions, experts' weight has been added to the proposed MCDM procedure. The experts' weight determination has been done according to the MABAC method and AIS. Also, the proposed reliable decision model has been extended under the grey numbers. Grey numbers as a valuable way for uncertainty consideration has been applied. Furthermore, the criteria' weight has been specified by using a development of the BWM (G-BWM) technique that was extended under the grey setting. Moreover, the authentication of the PM has been derived by comparing it with the TOPSIS method's results. In many practical situations, especially for conveyor selection based on the conflict criteria, existing a comprehensive and reliable method is necessary. In addition to a reliable and stable method for ranking of alternatives (conveyors), a weighting method for criteria and experts in real-world situations is very crucial because the weight of each criterion is different from each other, and the experts with diverse experience and knowledge do not have the same weight in the decision-making procedure. In real-world conditions, most decisions are made collectively. For this purpose, adding a method for experts' weight determination is vital. This paper provides a comprehensive approach with the higher different degree values over the well-known MCDM method (e.g., TOPSIS method) for raking and weighting. The data-driven MCDM approach can be enhanced the PM for defining the weight of experts or criteria. The PM can be utilized in various MCDM problems of practical situations, e.g., project selection, supplier selection, critical path determination, robot selection, risk analysis, knowledge management system evaluation, quality of life variables evaluation in senior design residences.

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