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# Applied decompositions of Malmquist, cost Malmquist, and allocation <br> Malmquist indices by considering changes in cost efficiency and technology 

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#### Abstract

So far, various decompositions of the Malmquist productivity growth index have been presented. Although factors such as efficiency, scale, and technology have already been examined, there is no factor measures productivity growth from a financial perspective by covering costs. The purpose of this article is to show the impact of cost efficiency changes as an important component on the productivity growth indices. This article evaluates the rate of productivity growth in the cost space and decomposes the Malmquist productivity growth index into components of cost efficiency and allocative efficiency. Then a similar Decomposition for the cost Malmquist index, and the allocation Malmquist index based on changes in cost efficiency and price effect is obtained. In the following, we obtain the relation between the Malmquist index, the Cost Malmquist index, and the allocation Malmquist index with changes in technology and cost efficiency. Then we model, and calculate the parsing factors of Malmquist indices related to decision-making units using data envelopment analysis and input distance functions. Finally, the data obtained from a real case study are modeled and compared the results of previous Malmquist indices with the new Malmquist indices and the preference of new decompositions has been analyzed.


Keywords: Data envelopment analysis, Malmquist index, cost Malmquist index, allocation Malmquist index, allocative efficiency, price effect

## 1- Introduction

In production theory, the production function is the function that produces the most outputs by different inputs. The lower surface of the production set is called the Production Possibility Set (P.P.S.). Parametric and nonparametric methods are used to estimate the production function. The most important parametric method is the Cobb-Douglas function, which is based on linear programming to calculate the output of production. Emerging nonparametric methods of calculating the production function can be related to Farrell (Farrell, 1957) that Assuming a constant return to scale (CRS) for two variables and calculates an estimate of the production function.

Then, a mathematical programming technique was defined to measure and compare the relative efficiency of decision-making units (DMUs) with multiple inputs and outputs, which operates in the form of a linear programming model based on CRS (Charnes, Cooper and Rhodes, 1978). This model is known as the CCR model, which has a mathematical programming structure due to the use of data envelopment analysis (DEA). Afterward, the CCR model was developed as a variable return to scale (VRS) or BCC model (Banker, Charnes and Cooper, 1984). Then, in 1999, profitability was modeled using data envelopment analysis (Camanho, Dyson, 1999).

[^0]DEA is an area of operations research that has many tools for calculating the efficiency of decisionmaking units (Ray, 1988), (Charnes et al. 1994), (Cooper, Lewin and Seiford, 1995), (Cooper, Seiford and Tone, 2000).
The Malmquist Index (MI) is another concept introduced by Malmquist in 1953. The total factor productivity growth (TFPG) is called the Malmquist index (Malmquist, 1953). This index is the most important index for measuring relative changes in productivity over a time when data envelopment analysis was used to calculate distance functions (Caves, Christensen and Divert, 1982), known as the CCD model.

In 1992, Fare, Grosskopf, Lindgren, and Roos decomposition (FGLR) of the Malmquist index using CRS technology was proposed, which included two components: efficiency change and technology change (Fare et al.1992).
In 1994 Fare, Grosskopf, Norris, and Zhang decomposition (FGNZ) of the Malmquist index was developed using both CRS and VRS technologies, and a three-part analysis of the index was obtained, which included pure efficiency change, scale efficiency change, and technology change (Fare et al. 1994). The method of calculating the change in scale efficiency is described in (Fare, Grosskopf, 1994). When manufacturers are looking to reduce costs and the price of inputs is known, Malmquist cost efficiency can be broken down into cost efficiency, allocative efficiency, and cost-technical changes that use cost components instead of input distance function components. (Maniadakis, Thanassoulis, 2004). In case the input prices are not clear and the upper and lower cost limits can be estimated for DMUs, the upper and lower limit estimation method is used to estimate the cost (Camanho, Dyson, 2005), which in case the input price levels are assumed to be known, using equality weight constraints with the price ratio, the cost efficiency model equivalent to the Farrell cost efficiency model is presented. Then Kuosmanen used upper and lower estimations to maximize cost efficiency (Kuosmanen, 2006). In 2015, Fang and Li developed mathematical programming models to calculate the upper and lower limits of cost efficiency, while prices are not unique (Fang, Li, 2015). After that, another model for calculating cost efficiency has introduced with the ability to use multiple outputs also inputs with specific outputs (Walheer, 2018). Given the prices of inputs (or outputs), in addition to the effect of technical efficiency, we would also like to consider the effect of costefficiency. In such a case, we measure the distance of each unit from the minimum cost limit (Thanassoulis, Silva, 2018).

Based on ideas, which use different technologies of DEA models to measure Malmquist index, in this article, by using COST technology instead of VRS technology, we find a new decomposition for Malmquist productivity growth, cost Malmquist (CM), and allocation Malmquist (AM) indices. The rest of the article is organized: Section 2 presents the technical background of the research. Section 3 presents decompositions of the MI, CM, and AM. Section 4 presents the calculations of the DEA indices and nonparametric planning models. In Section 5, a practical example for 40 branches of a country's leading banks located in the eastern part of Tehran, using the data of 2017 and 2018 as a real case study with the analysis of the results is obtained. It is worth mentioning that this bank has more than 1300 branches in 38 regions of the country.

## 2- Technical background

Suppose the production unit uses the inputs $x \epsilon R_{+}^{p}$ to produce the outputs $y \epsilon R_{+}^{q}$. We consider T technology:

$$
T=\left\{(x . y) \in R_{+}^{p} \times R_{+}^{q} \mid x \text { can produce } y\right\}
$$

$R_{+}^{p} \times R_{+}^{q} \subset T$ are all possible combinations of input-output values. Also, T is closed, convex, and bounded, and satisfies in the following condition (Fare, Primont, 1995), (Shephard, 1970):

$$
\begin{equation*}
(x . y) \in T, x^{\prime} \geq x, y^{\prime} \leq y \rightarrow\left(x^{\prime} . y^{\prime}\right) \in T \tag{1}
\end{equation*}
$$

The input and output sets in period t , are considered:
$L^{t}(y)=\left\{x \mid(x . y) \in T^{t}\right\}$
$P^{t}(x)=\left\{y \mid(x . y) \in T^{t}\right\}$
In this case, the input distance function is defined (Shephard, 1970):
$D_{i}^{t}(y, x)=\underset{\lambda}{\operatorname{Sup}}\left\{\lambda>0 \mid(x / \lambda, y) \in T^{t}\right\}$
Or equivalently
$D_{i}^{t}(y, x)=\underset{\lambda}{\operatorname{Sup}}\left\{\lambda>0 \mid(x / \lambda) \epsilon L^{t}(y)\right\}$
In fact, $D_{i}^{t}(y \cdot x)$ is the largest value on which the inputs $x^{t}$ can be divided and remain in $L^{t}(y)$. In other words, for any vector $x$, the vector $x / D_{i}^{t}(y \cdot x)$ is the smallest input vector on the ray passing through the origin and x can produce $y$.

For the input distance function with T technology we can write:
$D_{i}^{t}(y . x) \geq 1 \leftrightarrow(x . y) \in T$
We define the technology frontier points:
Isoq $L(y)=\{x \in L(y) \mid \lambda<1 \rightarrow \lambda \mathrm{x}$ not belong to $L(y)\}$
Technical efficiency (TE): The ability of the production unit to produce maximum outputs from a certain set of inputs is called technical efficiency. Its purpose is to use the minimum inputs to get maximum production outputs.
Suppose $x^{t}$ and $y^{t}$ are the input and output values of period t . We saw that $x^{t} / D_{i}^{t}\left(y^{t} \cdot x^{t}\right)$ is the smallest feasible input vector that can produce $y^{t}$. So we can write:
$I T E^{t}\left(y^{t} \cdot x^{t}\right)=\left\|x^{t} / D_{i}^{t}\left(y^{t} \cdot x^{t}\right)\right\| /\left\|x^{t}\right\|=\left(1 / D_{i}^{t}\left(y^{t} \cdot x^{t}\right)\right) \leq 1$
$T E C_{i}^{t . t+1}(y \cdot x)=D_{i}^{t}(y \cdot x) / D_{i}^{t+1}(y \cdot x)$
And in the case where $\operatorname{ITE}^{t}\left(x^{t} \cdot y^{t}\right)=1$ input is technically efficient.
Constant Return to Scale Technology: T technology in period t is called Constant Return
to Scale if $(x . y) \epsilon T^{t}, \lambda>0 \rightarrow(\lambda x . \lambda y) \epsilon T^{t}$
In other words, for every $\lambda>0$ we have. $T^{t}=\lambda T^{t}$.
An equivalent condition for constant return to scale technology is to have $L^{t}(\lambda y)=\lambda L^{t}(y)$ for every y and $\lambda>0$.

Cost function $C(\mathrm{y} . \mathrm{w})$ : For input prices $w \in R_{+}^{p}$, the minimum cost function for generating y is defined:
$C(\mathrm{y} . \mathrm{w})=\min _{\mathrm{x}}\{w x \mid x \in L(y)\}, w^{t} x^{t}=\sum_{n=1}^{p} w_{n}^{t} x_{n}^{t}$
Which the index n represents the nth input.
Given the convexity assumption, there is a dual relationship between the cost function and the input distance function (Fare and Primont, 1995):
$\mathrm{C}^{t}(\mathrm{y} \cdot \mathrm{w})=\min _{\mathrm{x}}\left\{w x / D_{i}^{t}(y \cdot x)\right\}$

$$
D_{i}^{t}(y \cdot x)=i n f_{w}\left\{w x / \mathrm{C}^{t}(\mathrm{y} \cdot \mathrm{w})\right\}
$$

Cost efficiency (CE): The ability of the production unit to produce maximum outputs by selecting the optimal set of inputs (inputs with the lowest cost at the efficiency frontier) is called cost efficiency or economic efficiency or overall efficiency (OE). In other words, cost efficiency is the ratio of the minimum cost of producing y to the actual cost of producing it, where w is the input price and is defined:
$O E(\mathrm{y} . \mathrm{x} \cdot \mathrm{w})=C(\mathrm{y} \cdot \mathrm{w}) / \mathrm{w} x$
The origins of the idea of CE can be traced back to Farrell in 1957, which developed many ideas under efficiency estimates (Farrell, 1957). We used the model (Camanho and Dyson, 2008) to calculate cost efficiency.

Allocative Efficiency (AE): The ability of the production unit to select the optimal set of inputs at the efficiency frontier, at the lowest cost, is called allocative efficiency which defined (Fare, Bogetoft, 2006):
$A E=C(y \cdot w) D_{i}(y \cdot x) / w x=O E(y \cdot x \cdot w) D_{i}(y \cdot x)$
$=\min _{x^{\prime}}\left\{w x^{\prime} * D_{i}(y . x) / w x \mid x^{\prime} \in L(y)\right\}$
$=\min x "\left\{w x " / w x \mid x " \epsilon D_{i}(y . x) L(y)\right\}$

In other words, if we propose x " for the production, the value required for production will be equal to $x^{\prime \prime} / D i(y . x)$. Allocative efficiency is earned by multiplying technical efficiency and cost efficiency. In fact, according to define cost-efficiency can be written:

$$
\begin{align*}
& O E=C E=\frac{C(y \cdot w)}{w x}=\left(\frac{C(y \cdot w) D_{i}(y \cdot x)}{w x}\right) *\left(\frac{1}{D_{i}(y \cdot x)}\right)=A E * T E  \tag{7}\\
& O E_{i}^{t}\left(y^{t} \cdot x^{t} \cdot w^{t}\right)=\frac{C^{t}\left(y^{t} \cdot w^{t}\right)}{w^{t} x^{t}} \leq \frac{w^{t} x^{t}}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)} * \frac{1}{w^{t} x^{t}}=\frac{1}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}
\end{align*}
$$

That is, cost efficiency will be at most equal to technical efficiency.
In other words, when production units use minimum inputs to obtain maximum outputs (technical efficiency) and find the lowest-cost inputs on the efficiency frontier (Allocative efficiency), it means they have been able to maximize Obtain outputs from the lowest cost inputs at the efficiency frontier (cost efficiency).

Malmquist Index (MI or IM): Suppose x and y are the inputs and outputs of the DMUs and the periods $t$ and $t+1$. In this case, the Malmquist productivity index based on the input-oriented distance functions is expressed:
$I M=\left[\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t}\left(y^{t} . x^{t}\right)} \frac{D_{i}^{t+1}\left(y^{t+1} . . x t+1\right.}{D_{i}^{t+1}\left(y^{t} . x^{t}\right)}\right]^{\frac{1}{2}}$
Where the symbol I represent the input distance function in the CRS space. Different decompositions of the above Malmquist index can be used, such as the FGLR decomposition, which separates the Malmquist index into two factors efficiency changes (EC) and technology change (TC) (Fare et al. 1992):
$I M=\frac{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}\left[\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)} \frac{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}\right]^{\frac{1}{2}}=T E C \times T C$
Where the efficiency change is obtained from the following relation:
$\operatorname{TEC}\left(y^{t} \cdot x^{t}\right)=\frac{T E^{t}\left(y^{t} \cdot x^{t}\right)}{T E^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}=\frac{1 / D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{1 / D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}=\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}$
The second decomposition, known as FGNZ, needs VRS efficiency, which provides pure technical efficiency (PTE). In sum, scale efficiency (SE) is calculated as the ratio of CRS and VRS efficiencies. The FGNZ decomposition will be (Fare et al. 1994):
$M I=\frac{D_{V R S}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{V R S}^{t}\left(y^{t} \cdot x^{t}\right)} \frac{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right) / D_{V R S}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right) / D_{V R S}^{t}\left(y^{t} \cdot x^{t}\right)}\left[\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)} \frac{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}\right]^{\frac{1}{2}}$

## $=P E C \times S E C \times T C$

Where the pure efficiency change (PEC) and the scale efficiency change (SEC) are obtained from the following equations:

$$
\begin{aligned}
& P E C=\frac{D_{V R S}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{V R S}^{t}\left(y^{t} \cdot x^{t}\right)} \\
& S E C=\frac{\mathrm{S}^{t}\left(\mathrm{x}^{t} \cdot \mathrm{y}^{t}\right)}{\mathrm{S}^{t+1}\left(\mathrm{x}^{t+1} \cdot \mathrm{y}^{t+1}\right)}=\frac{D_{V R S}^{t}\left(y^{t} \cdot x^{t}\right) / D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{D_{V R S}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right) / D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)} \\
& =\left[\frac{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right) / D_{V R S}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right) / D_{V R S}^{t}\left(y^{t} \cdot x^{t}\right)}\right]
\end{aligned}
$$

## 3- New decompositions of Malmquist productivity growth, cost Malmquist, and allocation Malmquist indices

## 3-1- New decomposition of Malmquist productivity index using cost-efficiency

Now, using COST as a new technology instead of VRS in FGLR, we are decomposing the new IM. We assume the distance function is input-oriented. However, the same conditions apply to the outputoriented distance function. We can write:

$$
\begin{equation*}
D^{t}(y \cdot x)=C^{t}(y \cdot w) * D^{t}(y \cdot x) / C^{t}(y \cdot \mathrm{w}) \tag{11}
\end{equation*}
$$

Considering the Malmquist productivity index, and by substituting the values of relation (10) in it we have:
$I M=\frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)} \frac{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}$
$* \frac{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)}{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}\left[\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right) D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right) D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}\right]^{\frac{1}{2}}$
$=\frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)}$
$* \frac{\left(\frac{w^{t} x^{t}}{c^{t}\left(y^{t} \cdot w^{t}\right) D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}\right)}{\left(\frac{w^{t+1} x^{t+1}}{c^{t+1}\left(y^{t+1} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}\right)}\left[\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right) D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right) D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}\right]^{\frac{1}{2}}$
$=C E C^{*}(A E C)^{-1} * T C$
The new factors are similar to EC and TC that VRS technology has been replaced by COST technology:
$C E C=P E C_{\text {COST }}=\frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)}$
$(A E C)^{-1}=S E C_{C R S} /$ COST $T=\left[\frac{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right) D_{i}^{t}\left(y^{t} . x^{t}\right)\right)}{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)\right)}\right]$
$T C=\left[\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right) D_{i}^{t}\left(y^{t} . x^{t}\right)}{D_{i}^{t+1}\left(y^{t+1} . x^{t+1}\right) D_{i}^{t+1}\left(y^{t} . x^{t}\right)}\right]^{\frac{l}{2}}$
The three-part decomposition of IM is expressed:
$I M=P E C_{\text {COST }} \times S E C_{\text {CRS }} /$ COST $-T C=C E C *(A E C)^{-1 * T C}$
That is, if we use the COST space instead of the VRS space and consider their scale changes in the CRS space, we need a new PEC and SEC equal to CEC and $(A E C)^{-1}$, respectively.
Therefore, we have achieved three-component decomposition for the input-oriented Malmquist index, the components of which use cost efficiency and allocative efficiency change. Allocative efficiency is at least as important as technical efficiency and mostly more than that and production units can improve their performance by changing input mix rather than decreasing the actual amount of inputs they use. It is important that when a unit changes its allocative efficiency over time that this should be reflected in the measurement of its productivity change. (Thanassoulis, Silva, 2018). Thus we have achieved decomposition that simultaneously examines the impact of changes in cost efficiency and allocative efficiency and technology on the Malmquist Productivity growth index.

## 3-2- New decomposition of cost Malmquist index using cost-efficiency

Now, using COST as the new technology instead of VRS in FGLR, we will decompose the new CM. We can write:
$\mathrm{CM}=\left[\frac{\left(w^{t} x^{t+1} / C^{t}\left(y^{t+1} \cdot w^{t}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)} \frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right.}{\left(w^{t+1} x^{t} / C^{t+1}\left(y^{t} \cdot w^{t+1}\right)\right)}\right]^{\frac{1}{2}}$
$\left.=\frac{\left(w^{t+1} x^{t+1} / c^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)} \llbracket \frac{\left(w^{t} x^{t} / /^{t}\left(y^{t} \cdot w^{t}\right)\right)}{\left(w^{t+1} x^{t} / C^{t+1}\left(y^{t} \cdot w^{t+1}\right)\right)} \frac{\left(w^{t} x^{t+1} / c^{t}\left(y^{t+1} \cdot w^{t}\right)\right)}{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}\right]^{1 / 2}$
$\left.=\frac{\left(w^{t+1} x^{t+1} / /^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} . w^{t}\right)\right)} \llbracket \frac{D_{t}^{t}\left(y^{t+1} . x^{t+1}\right) D_{i}^{t}\left(y^{t} . x^{t}\right)}{D_{i}^{t+1}\left(y^{t+1} . . x^{t+1}\right) D_{i}^{t+1}\left(y^{t} . x^{t}\right)}\right]^{\frac{1}{2}}$
$\left.* \llbracket \frac{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right) D_{i}^{t}\left(y^{t} \cdot x^{t}\right)\right)}{\left(w^{t+1} x^{t} / C^{t+1}\left(y^{t} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)\right)} \frac{\left(w^{t} x^{t+1} / c^{t}\left(y^{t+1} \cdot w^{t}\right) D_{D}^{t}\left(y^{t+1} \cdot x^{t+1}\right)\right)}{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)\right)}\right]^{1 / 2}$

## $=\mathrm{CEC} * \mathrm{TC} * \mathrm{PE}=\mathrm{CEC} * \mathrm{PE} * \mathrm{TC}$

Where we have:
$\mathrm{CEC}=\mathrm{OEC}=\frac{O E^{t}\left(y^{t} \cdot x^{t} \cdot w^{t}\right)}{O E^{t+1}\left(y^{t+1} \cdot x^{t+1} \cdot w^{t+1}\right)}=\frac{C^{t}\left(y^{t} \cdot w^{t}\right) /\left(w^{t} x^{t}\right.}{C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right) / w^{t+1} x^{t+1}}$
$=\frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)}$

This is the same conclusion reached in the (Thanassoulis, Silva, 2018) article in another way. That is, if we use COST space instead of VRS space, we need one (PEC) and (SEC) equal to CEC and Price Effect (PE), to have a new decomposition in COST space for the CM productivity index.

## 3-3- New decomposition of allocation Malmquist index using cost-efficiency

Now, using COST as a new technology instead of VRS in FGLR, let's decompose the new AM:
$\mathrm{AM}=\left[\frac{\left(w^{t} x^{t+1} / C^{t}\left(y^{t+1} \cdot w^{t}\right) D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right) D_{i}^{t}\left(y^{t} \cdot x^{t}\right)\right)} \frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)\right)}{\left(w^{t+1} x^{t} / C^{t+1}\left(y^{t} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)\right)}\right]^{\frac{1}{2}}$
$=\frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right) D_{i}^{t}\left(y^{t} \cdot x^{t}\right)\right)}$
$\left[\frac{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right) D_{i}^{t}\left(y^{t} \cdot x^{t}\right)\right)}{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)\right)} \frac{\left(w^{t} x^{t+1} / C^{t}\left(y^{t+1} \cdot w^{t}\right) D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)\right)}{\left(w^{t+1} x^{t} / C^{t+1}\left(y^{t} \cdot w^{t+1}\right) D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)\right)}\right]^{\frac{1}{2}}$
$\left.=\frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)} \frac{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)} \llbracket \frac{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)} \frac{D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}\right]^{\mathbf{1} / \mathbf{2}}$
$\llbracket \frac{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)}{\left(w^{t+1} x^{t} / C^{t+1}\left(y^{t} \cdot w^{t+1}\right)\right)} \frac{\left(w^{t} x^{t+1} / C^{t}\left(y^{t+1} \cdot w^{t}\right)\right)}{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)} \rrbracket^{1 / 2}$
$=\frac{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)}{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)} \frac{D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)} \llbracket \frac{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)} \frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)} \rrbracket^{\mathbf{1} / \mathbf{2}}$
$\llbracket \frac{\left(w^{t} x^{t} / C^{t}\left(y^{t} \cdot w^{t}\right)\right)}{\left(w^{t+1} x^{t} / C^{t+1}\left(y^{t} \cdot w^{t+1}\right)\right)} \frac{\left(w^{t} x^{t+1} / C^{t}\left(y^{t+1} \cdot w^{t}\right)\right)}{\left(w^{t+1} x^{t+1} / C^{t+1}\left(y^{t+1} \cdot w^{t+1}\right)\right)} \|^{\mathbf{1 / 2}}$
$=\operatorname{CEC}^{*}\left(\mathrm{CTC} * \frac{D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}\right) * \mathrm{TC}$
On the other hand, we can write:
$I M * T C=\left[\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t}\left(y^{t} \cdot x^{t}\right)} \frac{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}\right]^{\frac{1}{2}}\left[\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right) D_{i}^{t}\left(y^{t} \cdot x^{t}\right)}{D_{i}^{t+1}\left(y^{t+1} \cdot x^{t+1}\right) D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}\right]^{\frac{I}{2}}$
$=\frac{D_{i}^{t}\left(y^{t+1} \cdot x^{t+1}\right)}{D_{i}^{t+1}\left(y^{t} \cdot x^{t}\right)}$
So by placing in AM we have:
$A M=C E C *\left(C T C * \frac{1}{I M * T C}\right) * T C$
$=C E C *\left(\frac{C T C}{I M * T C}\right) * T C=C E C *\left(\frac{P E * T C}{I M * T C}\right) * T C=C E C *\left(\frac{P E}{I M}\right) * T C$
That is, if we use COST space instead of VRS space, we need a PEC and SEC equal to CECand $A(S E C)=\frac{P E}{I M}$, respectively, to have a new decomposition in CRS space for the AM index.
Considering the Malmquist indices IM, CM, and AM, and considering the CEC and TC components are common to all of them, we can write:
$\frac{I M}{(A E C)^{-1}}=\frac{C M}{P E}=\frac{A M}{P E *(I M)^{-1}}(=\mathrm{CEC} * \mathrm{TC})$

As a side effect we have relation:
$\frac{I M}{(A E C)^{-1}}=\frac{C M}{P E} \rightarrow C M=I M *(A E C) * P E=I M * A M$
The relationship between technical efficiency, cost efficiency, and allocative efficiency in (8) is also proved between their Malmquist indices.

It should be noted that (Thanassoulis, Silva, 2018) article only deals CM decomposition with cost efficiency component, which we have obtained in another way. We also reiterate that in this article we present IM, CM, and AM decompositions with cost efficiency component and then analyze the results and there is no article that parses IM and AM in the COST space using AE and PE.

## 4- Computational planning models of productivity indices components

Assuming the T technology is considered a fixed or variable return to scale, we use their corresponding distance functions $D_{i}^{t+l}\left(y_{p}^{t} \cdot x_{p}^{t}\right)$ and $D_{V R S}^{t+1}\left(y_{p}^{t} \cdot x_{p}^{t}\right)$.
Considering distance function, the following relationships can be expressed between the inputoriented distance function and DEA models with input-oriented CRS technology (Caves, Christensen and Divert, 1982). For instance, for $D_{i}^{t+l}\left(y_{p}^{t} \cdot x_{p}^{t}\right)$ we have the dual model of the following DEA fractional model:

$$
\begin{equation*}
\left[D_{i}^{t+1}\left(y_{p}^{t} \cdot x_{p}^{t}\right)\right]^{-1}=\min _{\phi, \lambda} \phi \tag{16}
\end{equation*}
$$

St.
$-y^{t}+Y^{t+1} \lambda \geq 0$
$\phi x^{t}-X^{t+1} \lambda \geq 0$ $\lambda \geq 0$

And similarly for $D_{V R S}^{t+l}\left(y_{p}^{t} \cdot x_{p}^{t}\right)$ we can write:
$\left[D_{V R S}^{t+l}\left(y_{p}^{t} \cdot x_{p}^{t}\right)\right]^{-1}=\min _{\phi, \lambda} \phi$
St.
$-y^{t}+Y^{t+1} \lambda \geq 0$
$\phi x^{t}-X^{t+1} \lambda \geq 0$
$\sum_{\lambda \geq 0} \lambda_{j}=1$

The resulting cost-efficiency model based on standard DEA formula is (Camanho, Dyson, 2008):
$C^{t}\left(y^{t} . w^{t}\right)=\min _{x . \lambda} w_{i} x_{i}^{*}$
St.

$$
\begin{aligned}
& -y_{i}+Y \lambda \geq 0 \\
& x_{i}^{*}-X \lambda \geq 0 \\
& N \lambda=1 \\
& \lambda \geq 0 \\
& \rightarrow C E=\frac{w_{i} x_{i}^{*}}{w_{i} x_{i}}
\end{aligned}
$$

## 5- Research method

## 5-1- An application in the Tehran banking industry

This section describes the theories, perspectives, and approaches to the problem and the model. Considering the article on achieving the products of bank branches (Paradi, Zhou, 2013), two inputs
and three outputs are considered: human resource and location index of branches are inputs, and deposits, facilities, and services are the outputs of this case study.
Human resource input has all the quantities and qualities related to the queuing staff in the branch. Also, the location input has all the quantities and qualities related to the physical location of the branch.
The output of branch deposits includes all types of methods of collecting cash by that branch. The weighted average of different types of investment accounts has been calculated according to the amounts and their number to obtain the deposit index, which we will use in the calculations related to the deposit index.
The output of the facility includes all the funds that have been paid by the branch in the form of various types of facilities, and like deposits, based on the importance of the type of facility, its weighted average is obtained under the facility index.
Finally, service output is an indicator that includes various fee services in the form of card issuance, types of guarantees and opening of visual and long-term documentary credit, foreign exchange transaction fees, and funds transfer fees by applying weighting coefficients by a branch to its customers is presented.
In all inputs and outputs, the planning department of the bank has performed a project for these indicators that we will use in our calculations.
The unit of measurement of costs is estimated at $1,000,000$. Other indicators have been normalized and all results have been rounded up to a maximum of four digits.

## 5-2- Statistical representation of the average values of inputs and outputs and cost indices

Now, we use the paper method to calculate the Malmquist index. The results of the Malmquist index, CM, AM, and their components using programming GAMS are shown in tables 2 to 6 (Brook et al. 1998).
The following table summarizes the statistical status of the average values of inputs and outputs and costs indices for the data of 40 decision-making units in 2017 and 2018 (for more details, see Appendix A.):

Table 1: Statistical representation of the average values of inputs and outputs and cost indices for 2017 and 2018

|  | $Y 1$ | $Y 2$ | $Y 3$ | $X 1$ | $X 2$ | $W 1$ | $W 2$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mean | 1020.6 | 1247.3 | 848.6 | 4043.20 | 674.8 | 1.7 | 3.8 |
| SD | 561.3 | 1672.7 | 607.1 | 1991.75 | 1150 | 0.8 | 0.6 |
| Min | 172.4 | 146.3 | 125.2 | 1535.43 | 963.3 | 1.0 | 2.4 |
| Max | 2708 | 9909 | 2232 | 9570.75 | 118.1 | 5.4 | 4.7 |

## 5-3- The efficiency of decision-making units

To show the importance of the presented in this article, we have calculated indices and their efficiencies during 2017 and 2018 (Appendix B). It can be seen the decision-making units in 2017 were not cost-effective because of technical inefficiency and allocative inefficiency ( $\mathrm{CE}=0.6723$ ). However, considering the value of $\mathrm{TE}=0.6733$, which was smaller than the value of $\mathrm{AE}=0.9922$, AE was less influential than TE to the CE inefficiency in 2017. A similar analysis is conducted for CE in 2018. Thus, considering the value of $\mathrm{TE}=0.6771$, which was smaller than the value of $\mathrm{AE}=$ 0.9928 , TE had a greater impact on CE inefficiency in 2018 than AE.

For example, given the average technical efficiency is $67.33 \%$, if the production units of the region reduce inputs by $22.67 \%$, they will reach the efficiency frontier.

On the other hand, in 2017, branches 12 and 26 were technically efficient, cost-efficient, and allocative efficient $(\mathrm{TE}=1, \mathrm{CE}=1, \mathrm{AE}=1)$ and in 2018, branches 7, 12, and 26 were technical efficient, cost-efficient, and allocative efficient. Also, 19 branches in 2017 and 19 branches in 2018 were allocative efficient, but they were not technically efficient or cost-efficient. Also, 3 branches in 2017 and 6 branches in 2018 show small allocative efficiency improvements ( $\mathrm{AE}=1.0001$ ).

In fact, on average, decision-making units have been able to allocate production inputs, given their cost. Besides, the low-level of cost efficiency (significant distance from the unit) in general shows the weakness of decision-making units in terms of both technical efficiency and allocative efficiency, where the impact of TE was less influential than AE to the CE in 2017.
Table 2 shows the average and standard deviation of TE, CE, and AE for 40 decision-making units in 2017 and 2018 separately (For more details, see Appendix B.). It can be seen the banking industry has been cost-inefficient in both years. The reason can be technical inefficiency and allocative inefficiency of the banking industry. As we can see, AE had less effect on CE than TE.

Table 2. Average values of CE, TE, and AE and their standard deviation for 2017 and 2018

|  | CE Mean | TE Mean | AE Mean | CE SD | TE SD | AE SD |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{2 0 1 7}$ | 0.6723 | 0.6733 | 0.9922 | 0.1478 | 0.1501 | 0.0195 |
| $\mathbf{2 0 1 8}$ | 0.6733 | 0.6771 | 0.9928 | 0.1411 | 0.1399 | 0.0202 |

## 5-4- Classification of components in terms of Malmquist index

Here we compare the method of calculating the Malmquist index of the FGNZ method with the method of calculating it in this article. The Malmquist index item values for 40 decision-making units with their geometric averages since 2017 and 2018 are given in Appendix C:

Table 3. Mean values of Malmquist productivity index components

| IM (VRS) |  |  |  |  | IM (COST) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DMU | $\begin{gathered} I M \\ (V R S) \end{gathered}$ | $P E C_{V R S}$ | $S E C_{\text {CRS } / \text { VRS }}$ | $T C_{C R S}$ | $\begin{gathered} I M \\ (\text { COST }) \end{gathered}$ | $\begin{aligned} & \text { CEC } \\ & =P E C_{C O S T} \end{aligned}$ | $\begin{aligned} & (A E C)^{-1} \\ & =S E C_{C R S} / \mathrm{COST} \end{aligned}$ | $T C_{\text {CRS }}$ |
| G-Mean | 0.8470 | 0.9597 | 1.0335 | 0.8540 | 0.8470 | 0.9912 | 1.0006 | 0.8540 |
| SD | 0.1261 | 0.0893 | 0.1086 | 0.1666 | 0.1261 | 0.1570 | 0.0282 | 0.1666 |

To examine the (12) matching the values of the geometric mean of the index and its components since 2017 and 2018, we can write:
$0.8470=0.9597 * 1.0335 * 0.8540=0.9912 * 1.0006 * 0.8540$
The values show that productivity growth $I M(V R S)=0.8470$, which is obtained using a geometric mean. Index analysis shows a shift in productivity toward a shift in efficiency $P E C_{V R S}$ and technology change $T C_{C R S}$ and scaling $S E C_{V R S / C R S}$ had a decreasing effect on productivity growth index. The SEC also had less impact on IM than the PEC and TC, because:
$0.0335=|1-S E C|<|1-P E C|=0.0403$
$0.0335=|1-S E C|<|1-T C|=0.1460$
On the other hand, according to equation (12), the productivity growth rate $I M($ COST $)=0.8470$, which shows the change in productivity change $C E C=P E C_{C O S T}=0.9912$ and the technology change $T C_{C R S}=0.8540$, and Scale $(A E C)^{-1}=S E C_{C R S} / \operatorname{COST}=1.0006$ had a decreasing effect on the productivity growth rate, which means that $A E C=0.9994$ and units had allocative efficiency in average. Thus the improvement of the index is not only due to the change in efficiency, but also the increase in cost efficiency change and the allocative efficiency change have taken place simultaneously.

An examination of the mean values of the data in Appendix B also shows the units had allocative efficiency in both 2017 and 2018. Also, according to equation (14), AEC had a greater effect on IM development than CEC and TC. Because:
$0.0006=|1-A E C|<|1-T C|=0.0088$
$0.0006=|1-A E C|<|1-C E C|=0.1460$
The effective causes in the progress of the IM productivity index are AEC, CEC, and finally TC. This highlights the importance of using a new parser that simultaneously uses efficiency change, cost efficiency change, and allocative efficiency change in calculating the IM productivity index.

## 5-5- Classification of components in terms of cost Malmquist productivity growth rate

Here we compare the method of calculating the previous Malmquist index with the method of calculating it in this article. The values of the Malmquist cost index components for 40 decisionmaking units with their geometric averages since 2017 and 2018 are given in Appendix D.

Table 4. Mean values of cost Malmquist productivity index components

| $C M$ |  |  |  | $C M(C O S T)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $D M U$ | $C M$ | $C E C$ | $C T C$ | $C M$ <br> $(C O S T)$ | $C E C$ <br> $=P E C_{\text {COST }}$ | $P E$ <br> $=S E C_{C R S} / C O S T$ | $T C_{C R S}$ |
| G-Mean | $\mathbf{1 . 1 0 2 8}$ | $\mathbf{0 . 9 9 1 2}$ | $\mathbf{1 . 1 1 2 6}$ | $\mathbf{1 . 1 0 2 8}$ | $\mathbf{0 . 9 9 1 2}$ | $\mathbf{1 . 3 0 2 8}$ | $\mathbf{0 . 8 5 4 0}$ |
| SD | $\mathbf{0 . 1 5 1 1}$ | $\mathbf{0 . 1 5 7 0}$ | $\mathbf{0 . 1 5 6 8}$ | $\mathbf{0 . 1 5 1 1}$ | $\mathbf{0 . 1 5 7 0}$ | $\mathbf{0 . 1 2 0 7}$ | $\mathbf{0 . 1 6 6 6}$ |

To examine the (13) corresponding to the values of the geometric mean of the index and its parts since 2017 and 2018, we can write:

$$
1.1028=0.9912 * 1.1126=0.9912 * 1.3028 * 0.8540
$$

The values show that the regress of cost efficiency $C M=1.1028$, which is obtained using the geometric mean. Analysis of the index shows that due to the increase in cost efficiency $\mathrm{CEC}=0.9912$, the regress of CM was because of the regress of cost technology $\mathrm{CTC}=1.1126$. The CTC also had a greater impact on the CM than the CEC , because:
$0.0088=|1-C E C|<|1-C T C|=0.1126$
On the other hand, according to equation $(13), C M(\operatorname{COST})=1.1028$, which shows that due to the increase in cost efficiency $C E C=P E C_{\text {COST }}=0.9912$ and the progress of technology $\mathrm{TC}=0.8540$, cost Malmquist decline was because of reduced price effect $P E=S E C_{C R S} /$ COST $=0.8540$. PE also had the greatest effect on CM compared with the other two parts.

## 5-6- Classification of items in allocation Malmquist productivity growth rate

Here we compare the method of calculating the previous allocation Malmquist index with the method of calculating it in this article. The values of the Malmquist index parts shared to 40 decisionmaking units with their geometric averages since 2017 and 2018 are given in Appendix E.

Table 5. Mean values of allocation Malmquist index productivity components

| $A M$ |  |  |  | $A M(C O S T)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $D M U$ | $A M$ | $A E C$ | $P E$ | $A M$ <br> $(C O S T)$ | $C E C$ <br> $=P E C_{\text {COST }}$ | $P E / I M$ <br> $=S E C_{C R S} / C O S T$ | $T C_{C R S}$ |
| G-Mean | $\mathbf{1 . 3 0 1 9}$ | $\mathbf{0 . 9 9 9 4}$ | $\mathbf{1 . 3 0 2 8}$ | $\mathbf{1 . 3 0 1 9}$ | $\mathbf{0 . 9 9 1 2}$ | $\mathbf{1 . 5 3 8 0}$ | $\mathbf{0 . 8 5 4 0}$ |
| SD | $\mathbf{0 . 1 3 0 9}$ | $\mathbf{0 . 0 2 8 1}$ | $\mathbf{0 . 1 2 0}$ | $\mathbf{0 . 1 3 0 9}$ | $\mathbf{0 . 1 5 7 0}$ | $\mathbf{0 . 3 1 9 3}$ | $\mathbf{0 . 1 6 6 6}$ |

To examine relation (14) we can write:
$1.3019=0.9994 * 1.3028=0.9912 * 1.5380 * 0.8540$
The values show that $A M=1.3019$ which is obtained using geometric mean. Analysis of the index shows the allocation Malmquist decline was due to the increase in cost efficiency CEC= 0.9912 due to regress of the price effect $\mathrm{PE}=1.3028$. The CEC also had less impact on AM than PE because: $0.0088=|1-C E C|<|1-P E|=0.3028$
On the other hand, according to equation (14), the analysis of the index shows the regress of the allocation Malmquist due to the increase in cost efficiency changes CEC $=0.9912$ and the progress of technology changes $\mathrm{TC}=0.8540$ was because of regressing of $\mathrm{PE} / \mathrm{IM}=1.5380$. In fact, due to the fact that $\mathrm{IM}=0.8470$, despite the fact that the units have improved productivity, due to the lack of proper use of the input price combination, AM regression has been created. The CEC also had less impact on AM than the other two components because:
$0.0088=|1-C E C|<|1-T C|=0.1460$
$0.0088=|1-C E C|<|1-P E / I M|=0.5380$

## 5-7- Classification of branches in technical, cost, and allocative efficiency

In this section, we analyze the values of the components of Malmquist indices, cost Malmquist, and allocation Malmquist according to the items of the analysis of this article. The values of indicators and components with their geometric average for 40 decision-making units since 2017 and 2018 are given in the table below (for more details, see Appendix F.):

Table 6. Mean values of productivity indices and their parts for 40 decision-making units

| IM Index |  |  | CM Index |  | AM Index |  | Common Factors |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $D M U$ | $I M$ | $1 / A E C$ | $C M$ | $P E$ | $A M$ | $P E / I M$ | $C E C$ | $T C$ |
| G-Mean | $\mathbf{0 . 8 4 7 0}$ | $\mathbf{1 . 0 0 0 6}$ | $\mathbf{1 . 1 0 2 8}$ | $\mathbf{1 . 3 0 2 8}$ | $\mathbf{1 . 3 0 1 9}$ | $\mathbf{1 . 5 3 8 0}$ | $\mathbf{0 . 9 9 1 2}$ | $\mathbf{0 . 8 5 4 0}$ |
| SD | $\mathbf{0 . 1 2 6 1}$ | $\mathbf{0 . 0 2 8 2}$ | $\mathbf{0 . 1 5 1 1}$ | $\mathbf{0 . 1 2 0 7}$ | $\mathbf{0 . 1 3 0 9}$ | $\mathbf{0 . 3 1 9 3}$ | $\mathbf{0 . 1 5 7 0}$ | $\mathbf{0 . 1 6 6 6}$ |

To examine the relation (15), it can be seen the geometric mean values of IM, CM, and AM indices related to 40 decision-making units in 2017 and 2018 have the following relations:
$\frac{I M}{(A E)^{-1}}=\frac{0.8470}{1.0006}=0.8465 \cdot \frac{C M}{P E}=\frac{1.1028}{1.3028}=0.8465 \cdot \frac{A M}{P E} / I M \quad=\frac{1.3019}{1.5380}=0.8465$
From the view of cost management, since the CM index was regressed in $2017(\mathrm{CM}=1.1028)$, due to an increase in cost efficiency $\mathrm{CEC}=0.9912$, was because of regress in cost technical change CTC $=1.1126$. The CEC also had less effect on the CM than the CTC.
On the other hand, according to equation (13), the reason the CM index declined in 2017 can be related to the decline of $\mathrm{PE}=1.3028$ because of the increase in cost efficiency $\mathrm{CEC}=0.9912$ and the progress of technology changes $\mathrm{TC}=0.8540$. Also, increasing cost efficiency compared with the other two items has less impact on CM regression because:
$0.0088=|1-C E C|<|1-T C|=0.146$
$0.0088=|1-C E C|<|1-P E|=0.3028$
Also, the effect caused by TC was less than the effect of PE on CM regression. That is, DMUs due to an increase in cost efficiency changes and technological advances, due to the lack of proper allocation of input prices could not are adequate CM .
In productivity size, considering equation (15), the CM regress due to IM progress was because of AM regress. In fact, AM had a greater effect on CM regress than IM.
From the view of production management, improving IM $=0.8470$ in 2017 and 2018 was because of the increase of $\mathrm{EC}=0.9918$ and $\mathrm{TC}=0.8540$. Also, EC had less impact on IM than TC. On the other hand, $\mathrm{PEC}=0.9597$ and $\mathrm{SEC}=1.0335$. Therefore, the PEC item has less impact on IM than the SEC.
From the view of allocation management, AM regressed in 2017 and 2018 due to increasing cost efficiency CEC $=0.9912$ and progress of technology changes $\mathrm{TC}=0.8540$ was because of $\mathrm{PE} / \mathrm{IM}=$ 1.5380 regress. In other words, the higher growth of the price effect compared with the productivity index has caused the CM to regress. Also, CEC has been less effective in AM reversal than the other two components and the most impact on AM was related to price effect.
According to Appendix (F), Branches 2, 7, 13, 16, 24, 27, 33, and 40 were cost-effective, and none of the branches was allocative efficient, and all units had regress of AM (AM> 1). Also for branches 7, 13 IM progresses had been more effective in progress CM than AM and in other branches AM progress had been more effective in progress than IM.
Also, 39 branches have progressed in productivity and for them IM <AM. For these branches, 8 branches had a progress in CM, and the regress of CM for other branches was because of the regress of AM.
We also have IM> AM for branch 25 and for this branch, the CM regress was because of the regress of IM and AM, but AM had less effect on CM than IM, because:
$0.0460=|1-A M|<|1-I M|=0.1642$

## 6- Conclusion

The cost information of our decision-making units led us to decompose the Malmquist productivity, cost Malmquist, and allocation Malmquist indices by considering changes in cost efficiency over twotime periods. The strategies defined on the input-oriented distance function in the Malmquist index led us to a new decomposition of this important index. In this paper, a new technology called COST is used to calculate distance functions. Considering that allocative efficiency is at least as important as technical efficiency and mostly more than that, the decompositions provide a more accurate analysis of the contribution of each factor of technology change, efficiency change, and cost efficiency change in productivity growth indices. When we use allocative efficiency instead of technical efficiency in decompositions, more accurate analyzes of the factors influencing growth or reducing the productivity growth index is obtained. Similar conditions apply to cost Malmquist and allocation Malmquist indices decomposed by factors such as price effect; because the price effect is inversely related to the allocative efficiency change. The new Malmquist indices were then applied to a case study to show the impact of new factors on efficiency rankings and productivity growth of decision-making units, and the results were analyzed. As a result, with the new decompositions, we can accurately determine whether the increase or decrease in productivity indices is due to changes in technical efficiency or the main reason is a change in allocative efficiency and price effect. As a future work offer, one can consider the role of revenue efficiency in Malmquist, cost Malmquist, and allocation Malmquist indices. Also, another technology such as revenue technology instead of COST and VRS could be used.

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## Appendix A:

Average values of inputs and outputs and price indices from 2017 to 2018

| DMU | Y1 | Y2 | Y3 | X1 | X2 | W1 | W2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 609.85 | 811.50 | 550.50 | 2952.49 | 1031.00 | 1.25 | 4.10 |
| 2 | 998.95 | 982.45 | 832.05 | 4459.63 | 1140.00 | 1.75 | 4.70 |
| 3 | 340.80 | 980.90 | 639.50 | 2135.65 | 696.20 | 1.05 | 2.50 |
| 4 | 1441.00 | 2914.50 | 894.40 | 6565.92 | 1090.00 | 2.60 | 4.50 |
| 5 | 441.90 | 1284.50 | 739.40 | 2722.38 | 707.40 | 1.15 | 2.50 |
| 6 | 1129.00 | 1420.50 | 800.25 | 4551.81 | 1097.00 | 1.75 | 4.50 |
| 7 | 3149.00 | 781.70 | 3315.50 | 8026.07 | 1150.00 | 3.10 | 4.70 |
| 8 | 1035.00 | 823.80 | 771.90 | 4357.19 | 1032.00 | 1.70 | 4.10 |
| 9 | 1154.00 | 1313.00 | 659.80 | 4021.79 | 1070.00 | 1.55 | 4.30 |
| 10 | 828.70 | 1244.00 | 720.15 | 4562.83 | 1022.00 | 1.80 | 4.10 |
| 11 | 1414.00 | 2358.00 | 1000.00 | 6907.83 | 1061.00 | 2.65 | 4.30 |
| 12 | 2454.00 | 10896.50 | 2178.00 | 11601.31 | 674.80 | 4.55 | 2.40 |
| 13 | 1124.05 | 615.70 | 1021.60 | 3977.59 | 1008.00 | 1.50 | 4.00 |
| 14 | 1001.90 | 1623.50 | 762.65 | 3754.29 | 1068.00 | 1.40 | 4.30 |
| 15 | 1015.00 | 1007.10 | 542.50 | 3866.61 | 1048.00 | 1.40 | 4.20 |
| 16 | 909.20 | 600.15 | 1396.50 | 3786.60 | 1010.00 | 1.40 | 4.00 |
| 17 | 1205.00 | 1933.00 | 814.95 | 5621.49 | 1092.00 | 2.25 | 4.40 |
| 18 | 1508.50 | 2364.50 | 1099.00 | 6647.73 | 1089.00 | 2.65 | 4.40 |
| 19 | 1810.50 | 797.40 | 2235.50 | 7265.68 | 1062.00 | 2.80 | 4.30 |
| 20 | 880.60 | 1207.00 | 720.20 | 4729.05 | 988.00 | 1.85 | 4.00 |
| 21 | 895.95 | 1018.90 | 637.90 | 2948.40 | 1006.00 | 1.20 | 4.00 |
| 22 | 1380.50 | 1416.50 | 965.45 | 5228.99 | 1029.00 | 2.05 | 4.10 |
| 23 | 1015.60 | 1197.50 | 625.40 | 2961.06 | 1023.00 | 1.20 | 4.10 |
| 24 | 820.25 | 841.20 | 847.00 | 3215.65 | 980.00 | 1.30 | 3.90 |
| 25 | 716.40 | 718.45 | 712.20 | 2742.78 | 938.70 | 1.20 | 3.70 |
| 26 | 1483.50 | 659.35 | 611.75 | 2537.32 | 968.60 | 1.15 | 3.80 |
| 27 | 1099.15 | 338.90 | 1305.50 | 4131.15 | 951.20 | 1.60 | 3.70 |
| 28 | 1062.20 | 1015.90 | 601.30 | 2968.56 | 927.90 | 1.20 | 3.60 |
| 29 | 1103.00 | 383.25 | 861.35 | 3440.38 | 919.80 | 1.35 | 3.60 |
| 30 | 711.35 | 1003.45 | 814.05 | 3079.24 | 924.40 | 1.25 | 3.60 |
| 31 | 987.25 | 987.10 | 906.05 | 4698.42 | 894.00 | 1.80 | 3.50 |
| 32 | 518.15 | 825.85 | 544.90 | 2791.18 | 918.40 | 1.20 | 3.60 |
| 33 | 984.55 | 438.80 | 442.85 | 2716.58 | 941.80 | 1.20 | 3.70 |
| 34 | 766.90 | 554.10 | 346.40 | 2610.33 | 925.50 | 1.15 | 3.60 |
| 35 | 340.40 | 448.90 | 287.85 | 2092.02 | 937.70 | 1.05 | 3.70 |
| 36 | 530.85 | 427.50 | 295.95 | 2195.39 | 852.00 | 1.05 | 3.20 |
| 37 | 890.95 | 670.90 | 533.80 | 3110.52 | 819.10 | 1.25 | 3.10 |
| 38 | 241.80 | 543.40 | 362.40 | 1958.45 | 813.20 | 1.05 | 3.00 |
| 39 | 626.50 | 236.85 | 173.95 | 2159.22 | 813.20 | 1.05 | 3.00 |
| 40 | 198.20 | 204.55 | 372.15 | 1628.37 | 813.20 | 1.00 | 3.00 |
| G-Mean | 889.2 | 904.8 | 722.1 | 3673.5 | 955.7 | 1.5 | 3.7 |
| SD | 553.9 | 1671.8 | 580.0 | 1985.5 | 118.1 | 0.7 | 0.6 |

## Appendix B:

OE, TE and AE values for 40 decision-making units and their average in 2017 and 2018

| $D M U$ | $T E(2017)$ | $C E(2017)$ | $A E(2017)$ | $T E(2018)$ | $C E(2018$ | $A E(2018)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.5962 | 0.5962 | 1.0000 | 0.5721 | 0.5721 | 1.0001 |
| 2 | 0.5007 | 0.4964 | 0.9914 | 0.6284 | 0.6284 | 1.0000 |
| 3 | 0.9878 | 0.9878 | 1.0000 | 0.7768 | 0.7768 | 1.0001 |
| 4 | 0.5978 | 0.5978 | 1.0000 | 0.7106 | 0.6858 | 0.9651 |
| 5 | 0.8919 | 0.8919 | 1.0000 | 0.7574 | 0.7574 | 1.0000 |
| 6 | 0.6335 | 0.6335 | 1.0000 | 0.6389 | 0.6389 | 1.0000 |
| 7 | 0.9552 | 0.8581 | 0.8983 | 1.0000 | 1.0000 | 1.0000 |
| 8 | 0.5677 | 0.5641 | 0.9936 | 0.5428 | 0.5391 | 0.9932 |
| 9 | 0.5982 | 0.5982 | 0.9999 | 0.7282 | 0.7282 | 1.0000 |
| 10 | 0.5442 | 0.5442 | 1.0000 | 0.5112 | 0.5112 | 0.9999 |
| 11 | 0.5744 | 0.5696 | 0.9916 | 0.6505 | 0.5787 | 0.8895 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 13 | 0.6442 | 0.6077 | 0.9433 | 0.7965 | 0.7965 | 1.0000 |
| 14 | 0.7023 | 0.7023 | 1.0000 | 0.8027 | 0.8027 | 0.9999 |
| 15 | 0.5484 | 0.5394 | 0.9837 | 0.6181 | 0.6181 | 1.0000 |
| 16 | 0.8926 | 0.8926 | 1.0000 | 0.7575 | 0.7575 | 1.0000 |
| 17 | 0.5674 | 0.5636 | 0.9933 | 0.6004 | 0.6004 | 0.9999 |
| 18 | 0.6268 | 0.6137 | 0.9791 | 0.6635 | 0.6343 | 0.9560 |
| 19 | 0.7687 | 0.7687 | 0.9999 | 0.6552 | 0.6347 | 0.9687 |
| 20 | 0.5228 | 0.5198 | 0.9944 | 0.5060 | 0.5060 | 1.0001 |
| 21 | 0.7313 | 0.7313 | 1.0000 | 0.7713 | 0.7713 | 1.0001 |
| 22 | 0.6204 | 0.5918 | 0.9539 | 0.6933 | 0.6679 | 0.9634 |
| 23 | 0.7966 | 0.7966 | 1.0000 | 0.8418 | 0.8418 | 1.0000 |
| 24 | 0.6995 | 0.6995 | 1.0000 | 0.7336 | 0.7336 | 1.0000 |
| 25 | 0.6738 | 0.6738 | 1.0000 | 0.7721 | 0.7721 | 1.0000 |
| 26 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 27 | 0.7800 | 0.7800 | 1.0000 | 0.6512 | 0.6512 | 1.0000 |
| 28 | 0.7672 | 0.7672 | 1.0000 | 0.8141 | 0.8141 | 1.0001 |
| 29 | 0.7717 | 0.7717 | 1.0000 | 0.6385 | 0.6347 | 0.9940 |
| 30 | 0.7634 | 0.7634 | 1.0000 | 0.7250 | 0.7250 | 1.0000 |
| 31 | 0.5642 | 0.5617 | 0.9956 | 0.5671 | 0.5593 | 0.9862 |
| 32 | 0.5618 | 0.5618 | 0.9999 | 0.6177 | 0.6177 | 1.0000 |
| 33 | 0.6436 | 0.6385 | 0.9920 | 0.6692 | 0.6665 | 0.9960 |
| 34 | 0.6311 | 0.6311 | 1.0001 | 0.5535 | 0.5535 | 1.0000 |
| 35 | 0.4847 | 0.4847 | 1.0000 | 0.4183 | 0.4183 | 1.0001 |
| 36 | 0.5402 | 0.5402 | 1.0001 | 0.5004 | 0.5004 | 1.0000 |
| 37 | 0.6105 | 0.5959 | 0.9761 | 0.6219 | 0.6219 | 1.0000 |
| 38 | 0.5468 | 0.5468 | 1.0001 | 0.5129 | 0.5129 | 1.0000 |
| 39 | 0.6345 | 0.6345 | 1.0000 | 0.4315 | 0.4315 | 1.0000 |
| 40 | 0.3887 | 0.3887 | 1.0000 | 0.6325 | 0.6325 | 1.0001 |
| $\boldsymbol{G}-\boldsymbol{M e a n}$ | $\mathbf{0 . 6 5 7 9}$ | $\mathbf{0 . 6 5 2 6}$ | $\mathbf{0 . 9 9 2 0}$ | $\mathbf{0 . 6 6 3 3}$ | $\mathbf{0 . 6 5 8 4}$ | $\mathbf{0 . 9 9 2 6}$ |
| $\boldsymbol{S D}$ | $\mathbf{0 . 1 5 0 1}$ | $\mathbf{0 . 1 4 7 8}$ | $\mathbf{0 . 0 1 9 5}$ | $\mathbf{0 . 1 3 9 9}$ | $\mathbf{0 . 1 4 1 1}$ | $\mathbf{0 . 0 2 0 2}$ |
|  |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |

## Appendix C:

Values of Malmquist Productivity Index components

| IM (VRS) |  |  |  |  | IM (COST) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DMU | $\begin{gathered} I M \\ (V R S) \end{gathered}$ | $P E C_{V R S}$ | $S E C_{\text {CRS }} /{ }_{\text {VRS }}$ | $T C_{C R S}$ | $\begin{array}{c\|} \hline I M \\ (C O S T) \end{array}$ | $\begin{aligned} & C E C \\ & =P E C_{\text {COST }} \end{aligned}$ | $\begin{aligned} & (A E C)^{-1} \\ & =S E C_{C R S} / \text { COST } \end{aligned}$ | $T C_{C R S}$ |
| 1 | 0.8012 | 1.0135 | 1.0282 | 0.7689 | 0.8012 | 1.0421 | 1.0001 | 0.7689 |
| 2 | 0.7755 | 0.8194 | 0.9723 | 0.9733 | 0.7755 | 0.7899 | 1.0087 | 0.9733 |
| 3 | 0.7759 | 1.0000 | 1.2716 | 0.6102 | 0.7759 | 1.2715 | 1.0001 | 0.6102 |
| 4 | 0.8648 | 0.8635 | 0.9742 | 1.0280 | 0.8648 | 0.8716 | 0.9651 | 1.0280 |
| 5 | 0.9574 | 0.9485 | 1.2416 | 0.8130 | 0.9574 | 1.1776 | 1.0000 | 0.8130 |
| 6 | 0.9316 | 0.9028 | 1.0983 | 0.9395 | 0.9316 | 0.9916 | 1.0000 | 0.9395 |
| 7 | 0.8564 | 0.9600 | 0.9950 | 0.8966 | 0.8564 | 0.8581 | 1.1132 | 0.8966 |
| 8 | 0.9390 | 0.9559 | 1.0941 | 0.8979 | 0.9390 | 1.0463 | 0.9996 | 0.8979 |
| 9 | 0.8526 | 0.8608 | 0.9544 | 1.0378 | 0.8526 | 0.8214 | 1.0001 | 1.0378 |
| 10 | 0.8388 | 0.9538 | 1.1161 | 0.7879 | 0.8388 | 1.0646 | 0.9999 | 0.7879 |
| 11 | 0.8446 | 0.8857 | 0.9970 | 0.9565 | 0.8446 | 0.9843 | 0.8971 | 0.9565 |
| 12 | 0.9874 | 1.0000 | 1.0000 | 0.9874 | 0.9874 | 1.0000 | 1.0000 | 0.9874 |
| 13 | 0.6744 | 0.8222 | 0.9837 | 0.8339 | 0.6744 | 0.7630 | 1.0601 | 0.8339 |
| 14 | 0.9352 | 0.8539 | 1.0246 | 1.0689 | 0.9352 | 0.8750 | 0.9999 | 1.0689 |
| 15 | 1.0522 | 0.8800 | 1.0082 | 1.1859 | 1.0522 | 0.8727 | 1.0166 | 1.1859 |
| 16 | 0.7957 | 1.0909 | 1.0802 | 0.6753 | 0.7957 | 1.1784 | 0.9999 | 0.6753 |
| 17 | 1.0002 | 0.8824 | 1.0710 | 1.0583 | 1.0002 | 0.9388 | 1.0067 | 1.0583 |
| 18 | 1.0538 | 0.9155 | 1.0319 | 1.1155 | 1.0538 | 0.9674 | 0.9765 | 1.1155 |
| 19 | 1.0567 | 1.0519 | 1.1153 | 0.9007 | 1.0567 | 1.2111 | 0.9687 | 0.9007 |
| 20 | 0.9335 | 0.9394 | 1.0999 | 0.9035 | 0.9335 | 1.0273 | 1.0058 | 0.9035 |
| 21 | 0.9377 | 0.9318 | 1.0175 | 0.9890 | 0.9377 | 0.9481 | 1.0000 | 0.9890 |
| 22 | 0.9494 | 0.8933 | 1.0017 | 1.0610 | 0.9494 | 0.8860 | 1.0099 | 1.0610 |
| 23 | 0.8974 | 0.9677 | 0.9778 | 0.9484 | 0.8974 | 0.9463 | 1.0000 | 0.9484 |
| 24 | 0.7142 | 0.8953 | 1.0650 | 0.7490 | 0.7142 | 0.9536 | 0.9999 | 0.7490 |
| 25 | 1.1642 | 0.8191 | 1.0654 | 1.3341 | 1.1642 | 0.8727 | 1.0000 | 1.3341 |
| 26 | 0.8514 | 1.0000 | 1.0000 | 0.8514 | 0.8514 | 1.0000 | 1.0000 | 0.8514 |
| 27 | 0.7115 | 1.0843 | 1.1046 | 0.5940 | 0.7115 | 1.1977 | 1.0001 | 0.5940 |
| 28 | 0.6865 | 0.9778 | 0.9638 | 0.7285 | 0.6865 | 0.9424 | 1.0000 | 0.7285 |
| 29 | 0.8815 | 1.0750 | 1.1243 | 0.7293 | 0.8815 | 1.2159 | 0.9940 | 0.7293 |
| 30 | 0.7619 | 0.9419 | 1.1180 | 0.7236 | 0.7619 | 1.0529 | 1.0000 | 0.7236 |
| 31 | 0.7588 | 0.9452 | 1.0526 | 0.7627 | 0.7588 | 1.0043 | 0.9906 | 0.7627 |
| 32 | 0.8135 | 0.9259 | 0.9823 | 0.8944 | 0.8135 | 0.9094 | 1.0001 | 0.8944 |
| 33 | 0.7257 | 1.0118 | 0.9506 | 0.7546 | 0.7257 | 0.9579 | 1.0040 | 0.7546 |
| 34 | 0.9000 | 1.0361 | 1.1004 | 0.7894 | 0.9000 | 1.1403 | 0.9999 | 0.7894 |
| 35 | 0.8480 | 1.1829 | 0.9796 | 0.7318 | 0.8480 | 1.1587 | 1.0001 | 0.7318 |
| 36 | 0.6548 | 1.1494 | 0.9392 | 0.6066 | 0.6548 | 1.0796 | 0.9999 | 0.6066 |
| 37 | 0.8215 | 0.9762 | 1.0056 | 0.8368 | 0.8215 | 0.9582 | 1.0245 | 0.8368 |
| 38 | 0.8621 | 1.0106 | 1.0549 | 0.8086 | 0.8621 | 1.0662 | 0.9999 | 0.8086 |
| 39 | 0.8714 | 1.1236 | 1.3087 | 0.5926 | 0.8714 | 1.4705 | 1.0000 | 0.5926 |
| 40 | 0.5207 | 1.0000 | 0.6145 | 0.8473 | 0.5207 | 0.6145 | 1.0001 | 0.8473 |
| G-Mean | 0.8470 | 0.9597 | 1.0335 | 0.8540 | 0.8470 | 0.9912 | 1.0006 | 0.8540 |
| SD | 0.1261 | 0.0893 | 0.1086 | 0.1666 | 0.1261 | 0.1570 | 0.0282 | 0.1666 |

## Appendix D:

Average values of Malmquist cost-effectiveness index components

| CM |  |  |  | CM (COST) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DMU | CM | CEC | CTC | $\begin{gathered} C M \\ (\text { COST }) \end{gathered}$ | $\begin{aligned} & C E C \\ & =P E C_{C O S T} \end{aligned}$ | $\begin{aligned} & P E \\ & =S E C_{C R S} / \text { COST } \end{aligned}$ | $T C_{\text {CRS }}$ |
| 1 | 1.1515 | 1.0421 | 1.1050 | 1.1515 | 1.0421 | 1.4372 | 0.7689 |
| 2 | 0.9698 | 0.7899 | 1.2278 | 0.9698 | 0.7899 | 1.2614 | 0.9733 |
| 3 | 1.1620 | 1.2715 | 0.9139 | 1.1620 | 1.2715 | 1.4977 | 0.6102 |
| 4 | 1.1290 | 0.8716 | 1.2952 | 1.1290 | 0.8716 | 1.2600 | 1.0280 |
| 5 | 1.2682 | 1.1776 | 1.0769 | 1.2682 | 1.1776 | 1.3245 | 0.8130 |
| 6 | 1.2099 | 0.9916 | 1.2201 | 1.2099 | 0.9916 | 1.2987 | 0.9395 |
| 7 | 0.8593 | 0.8581 | 1.0014 | 0.8593 | 0.8581 | 1.1169 | 0.8966 |
| 8 | 1.2330 | 1.0463 | 1.1784 | 1.2330 | 1.0463 | 1.3125 | 0.8979 |
| 9 | 1.0566 | 0.8214 | 1.2863 | 1.0566 | 0.8214 | 1.2394 | 1.0378 |
| 10 | 1.2069 | 1.0646 | 1.1336 | 1.2069 | 1.0646 | 1.4388 | 0.7879 |
| 11 | 1.2484 | 0.9843 | 1.2683 | 1.2484 | 0.9843 | 1.3260 | 0.9565 |
| 12 | 1.2512 | 1.0000 | 1.2512 | 1.2512 | 1.0000 | 1.2671 | 0.9874 |
| 13 | 0.7238 | 0.7630 | 0.9487 | 0.7238 | 0.7630 | 1.1377 | 0.8339 |
| 14 | 1.1679 | 0.8750 | 1.3348 | 1.1679 | 0.8750 | 1.2488 | 1.0689 |
| 15 | 1.1783 | 0.8727 | 1.3501 | 1.1783 | 0.8727 | 1.1384 | 1.1859 |
| 16 | 0.9707 | 1.1784 | 0.8237 | 0.9707 | 1.1784 | 1.2198 | 0.6753 |
| 17 | 1.2364 | 0.9388 | 1.3170 | 1.2364 | 0.9388 | 1.2444 | 1.0583 |
| 18 | 1.3199 | 0.9674 | 1.3644 | 1.3199 | 0.9674 | 1.2231 | 1.1155 |
| 19 | 1.1987 | 1.2111 | 0.9898 | 1.1987 | 1.2111 | 1.0990 | 0.9007 |
| 20 | 1.2446 | 1.0273 | 1.2116 | 1.2446 | 1.0273 | 1.3410 | 0.9035 |
| 21 | 1.2041 | 0.9481 | 1.2700 | 1.2041 | 0.9481 | 1.2841 | 0.9890 |
| 22 | 1.1623 | 0.8860 | 1.3117 | 1.1623 | 0.8860 | 1.2364 | 1.0610 |
| 23 | 1.1826 | 0.9463 | 1.2498 | 1.1826 | 0.9463 | 1.3178 | 0.9484 |
| 24 | 0.9926 | 0.9536 | 1.0409 | 0.9926 | 0.9536 | 1.3897 | 0.7490 |
| 25 | 1.2177 | 0.8727 | 1.3953 | 1.2177 | 0.8727 | 1.0459 | 1.3341 |
| 26 | 1.0719 | 1.0000 | 1.0719 | 1.0719 | 1.0000 | 1.2591 | 0.8514 |
| 27 | 0.9508 | 1.1977 | 0.7938 | 0.9508 | 1.1977 | 1.3364 | 0.5940 |
| 28 | 1.0115 | 0.9424 | 1.0734 | 1.0115 | 0.9424 | 1.4734 | 0.7285 |
| 29 | 1.2117 | 1.2159 | 0.9966 | 1.2117 | 1.2159 | 1.3664 | 0.7293 |
| 30 | 1.0870 | 1.0529 | 1.0324 | 1.0870 | 1.0529 | 1.4267 | 0.7236 |
| 31 | 1.0740 | 1.0043 | 1.0694 | 1.0740 | 1.0043 | 1.4021 | 0.7627 |
| 32 | 1.0751 | 0.9094 | 1.1822 | 1.0751 | 0.9094 | 1.3217 | 0.8944 |
| 33 | 0.9881 | 0.9579 | 1.0315 | 0.9881 | 0.9579 | 1.3670 | 0.7546 |
| 34 | 1.2015 | 1.1403 | 1.0536 | 1.2015 | 1.1403 | 1.3348 | 0.7894 |
| 35 | 1.2610 | 1.1587 | 1.0883 | 1.2610 | 1.1587 | 1.4871 | 0.7318 |
| 36 | 1.0551 | 1.0796 | 0.9773 | 1.0551 | 1.0796 | 1.6111 | 0.6066 |
| 37 | 1.1059 | 0.9582 | 1.1542 | 1.1059 | 0.9582 | 1.3792 | 0.8368 |
| 38 | 1.1176 | 1.0662 | 1.0482 | 1.1176 | 1.0662 | 1.2963 | 0.8086 |
| 39 | 1.2579 | 1.4705 | 0.8554 | 1.2579 | 1.4705 | 1.4436 | 0.5926 |
| 40 | 0.5837 | 0.6145 | 0.9499 | 0.5837 | 0.6145 | 1.1210 | 0.8473 |
| G-Mean | 1.1028 | 0.9912 | 1.1126 | 1.1028 | 0.9912 | 1.3028 | 0.8540 |
| SD | 0.1511 | 0.1570 | 0.1568 | 0.1511 | 0.1570 | 0.1207 | 0.1666 |

## Appendix E:

Values of Malmquist Productivity Index components

| AM |  |  |  | AM (COST) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DMU | AM | AEC | PE | $\begin{gathered} A M \\ (\text { COST }) \end{gathered}$ | $\begin{aligned} & \text { CEC } \\ & =P E C_{C O S T} \end{aligned}$ | $\begin{aligned} & P E / I M \\ & =S E C_{C R S} / \text { COST } \end{aligned}$ | $T C_{C R S}$ |
| 1 | 1.4371 | 0.9999 | 1.4372 | 1.4371 | 1.0421 | 1.7937 | 0.7689 |
| 2 | 1.2505 | 0.9913 | 1.2614 | 1.2505 | 0.7899 | 1.6265 | 0.9733 |
| 3 | 1.4976 | 0.9999 | 1.4977 | 1.4976 | 1.2715 | 1.9302 | 0.6102 |
| 4 | 1.3055 | 1.0361 | 1.2600 | 1.3055 | 0.8716 | 1.4570 | 1.0280 |
| 5 | 1.3246 | 1.0000 | 1.3245 | 1.3246 | 1.1776 | 1.3835 | 0.8130 |
| 6 | 1.2987 | 1.0000 | 1.2987 | 1.2987 | 0.9916 | 1.3941 | 0.9395 |
| 7 | 1.0033 | 0.8983 | 1.1169 | 1.0033 | 0.8581 | 1.3042 | 0.8966 |
| 8 | 1.3130 | 1.0004 | 1.3125 | 1.3130 | 1.0463 | 1.3977 | 0.8979 |
| 9 | 1.2393 | 0.9999 | 1.2394 | 1.2393 | 0.8214 | 1.4537 | 1.0378 |
| 10 | 1.4389 | 1.0001 | 1.4388 | 1.4389 | 1.0646 | 1.7154 | 0.7879 |
| 11 | 1.4781 | 1.1147 | 1.3260 | 1.4781 | 0.9843 | 1.5700 | 0.9565 |
| 12 | 1.2671 | 1.0000 | 1.2671 | 1.2671 | 1.0000 | 1.2833 | 0.9874 |
| 13 | 1.0732 | 0.9433 | 1.1377 | 1.0732 | 0.7630 | 1.6869 | 0.8339 |
| 14 | 1.2489 | 1.0001 | 1.2488 | 1.2489 | 0.8750 | 1.3353 | 1.0689 |
| 15 | 1.1198 | 0.9836 | 1.1384 | 1.1198 | 0.8727 | 1.0820 | 1.1859 |
| 16 | 1.2199 | 1.0001 | 1.2198 | 1.2199 | 1.1784 | 1.5331 | 0.6753 |
| 17 | 1.2362 | 0.9934 | 1.2444 | 1.2362 | 0.9388 | 1.2442 | 1.0583 |
| 18 | 1.2526 | 1.0241 | 1.2231 | 1.2526 | 0.9674 | 1.1607 | 1.1155 |
| 19 | 1.1344 | 1.0323 | 1.0990 | 1.1344 | 1.2111 | 1.0400 | 0.9007 |
| 20 | 1.3333 | 0.9943 | 1.3410 | 1.3333 | 1.0273 | 1.4365 | 0.9035 |
| 21 | 1.2840 | 1.0000 | 1.2841 | 1.2840 | 0.9481 | 1.3693 | 0.9890 |
| 22 | 1.2242 | 0.9902 | 1.2364 | 1.2242 | 0.8860 | 1.3022 | 1.0610 |
| 23 | 1.3178 | 1.0000 | 1.3178 | 1.3178 | 0.9463 | 1.4684 | 0.9484 |
| 24 | 1.3898 | 1.0001 | 1.3897 | 1.3898 | 0.9536 | 1.9458 | 0.7490 |
| 25 | 1.0460 | 1.0000 | 1.0459 | 1.0460 | 0.8727 | 0.8984 | 1.3341 |
| 26 | 1.2591 | 1.0000 | 1.2591 | 1.2591 | 1.0000 | 1.4789 | 0.8514 |
| 27 | 1.3363 | 0.9999 | 1.3364 | 1.3363 | 1.1977 | 1.8783 | 0.5940 |
| 28 | 1.4734 | 1.0000 | 1.4734 | 1.4734 | 0.9424 | 2.1461 | 0.7285 |
| 29 | 1.3746 | 1.0060 | 1.3664 | 1.3746 | 1.2159 | 1.5502 | 0.7293 |
| 30 | 1.4266 | 1.0000 | 1.4267 | 1.4266 | 1.0529 | 1.8724 | 0.7236 |
| 31 | 1.4153 | 1.0095 | 1.4021 | 1.4153 | 1.0043 | 1.8478 | 0.7627 |
| 32 | 1.3216 | 0.9999 | 1.3217 | 1.3216 | 0.9094 | 1.6248 | 0.8944 |
| 33 | 1.3616 | 0.9960 | 1.3670 | 1.3616 | 0.9579 | 1.8837 | 0.7546 |
| 34 | 1.3349 | 1.0001 | 1.3348 | 1.3349 | 1.1403 | 1.4830 | 0.7894 |
| 35 | 1.4871 | 0.9999 | 1.4871 | 1.4871 | 1.1587 | 1.7538 | 0.7318 |
| 36 | 1.6112 | 1.0001 | 1.6111 | 1.6112 | 1.0796 | 2.4602 | 0.6066 |
| 37 | 1.3463 | 0.9761 | 1.3792 | 1.3463 | 0.9582 | 1.6789 | 0.8368 |
| 38 | 1.2965 | 1.0001 | 1.2963 | 1.2965 | 1.0662 | 1.5038 | 0.8086 |
| 39 | 1.4436 | 1.0000 | 1.4436 | 1.4436 | 1.4705 | 1.6567 | 0.5926 |
| 40 | 1.1210 | 0.9999 | 1.1210 | 1.1210 | 0.6145 | 2.1528 | 0.8473 |
| G-Mean | 1.3019 | 0.9994 | 1.3028 | 1.3019 | 0.9912 | 1.5380 | 0.8540 |
| SD | 0.1309 | 0.0281 | 0.1207 | 0.1309 | 0.1570 | 0.3193 | 0.1666 |

## Appendix F:

Values of IM, CM, and AM and their components for 40 decision-making units

| IM Index |  |  | CM Index |  | AM Index |  | Common Factors |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| DMU | IM | 1/AEC | CM | PE | AM | PE/IM | CEC | TC |
| 1 | 0.8012 | 1.0001 | 1.1515 | 1.4372 | 1.4371 | 1.7937 | 1.0421 | 0.7689 |
| 2 | 0.7755 | 1.0087 | 0.9698 | 1.2614 | 1.2505 | 1.6265 | 0.7899 | 0.9733 |
| 3 | 0.7759 | 1.0001 | 1.1620 | 1.4977 | 1.4976 | 1.9302 | 1.2715 | 0.6102 |
| 4 | 0.8648 | 0.9651 | 1.1290 | 1.2600 | 1.3055 | 1.4570 | 0.8716 | 1.0280 |
| 5 | 0.9574 | 1.0000 | 1.2682 | 1.3245 | 1.3246 | 1.3835 | 1.1776 | 0.8130 |
| 6 | 0.9316 | 1.0000 | 1.2099 | 1.2987 | 1.2987 | 1.3941 | 0.9916 | 0.9395 |
| 7 | 0.8564 | 1.1132 | 0.8593 | 1.1169 | 1.0033 | 1.3042 | 0.8581 | 0.8966 |
| 8 | 0.9390 | 0.9996 | 1.2330 | 1.3125 | 1.3130 | 1.3977 | 1.0463 | 0.8979 |
| 9 | 0.8526 | 1.0001 | 1.0566 | 1.2394 | 1.2393 | 1.4537 | 0.8214 | 1.0378 |
| 10 | 0.8388 | 0.9999 | 1.2069 | 1.4388 | 1.4389 | 1.7154 | 1.0646 | 0.7879 |
| 11 | 0.8446 | 0.8971 | 1.2484 | 1.3260 | 1.4781 | 1.5700 | 0.9843 | 0.9565 |
| 12 | 0.9874 | 1.0000 | 1.2512 | 1.2671 | 1.2671 | 1.2833 | 1.0000 | 0.9874 |
| 13 | 0.6744 | 1.0601 | 0.7238 | 1.1377 | 1.0732 | 1.6869 | 0.7630 | 0.8339 |
| 14 | 0.9352 | 0.9999 | 1.1679 | 1.2488 | 1.2489 | 1.3353 | 0.8750 | 1.0689 |
| 15 | 1.0522 | 1.0166 | 1.1783 | 1.1384 | 1.1198 | 1.0820 | 0.8727 | 1.1859 |
| 16 | 0.7957 | 0.9999 | 0.9707 | 1.2198 | 1.2199 | 1.5331 | 1.1784 | 0.6753 |
| 17 | 1.0002 | 1.0067 | 1.2364 | 1.2444 | 1.2362 | 1.2442 | 0.9388 | 1.0583 |
| 18 | 1.0538 | 0.9765 | 1.3199 | 1.2231 | 1.2526 | 1.1607 | 0.9674 | 1.1155 |
| 19 | 1.0567 | 0.9687 | 1.1987 | 1.0990 | 1.1344 | 1.0400 | 1.2111 | 0.9007 |
| 20 | 0.9335 | 1.0058 | 1.2446 | 1.3410 | 1.3333 | 1.4365 | 1.0273 | 0.9035 |
| 21 | 0.9377 | 1.0000 | 1.2041 | 1.2841 | 1.2840 | 1.3693 | 0.9481 | 0.9890 |
| 22 | 0.9494 | 1.0099 | 1.1623 | 1.2364 | 1.2242 | 1.3022 | 0.8860 | 1.0610 |
| 23 | 0.8974 | 1.0000 | 1.1826 | 1.3178 | 1.3178 | 1.4684 | 0.9463 | 0.9484 |
| 24 | 0.7142 | 0.9999 | 0.9926 | 1.3897 | 1.3898 | 1.9458 | 0.9536 | 0.7490 |
| 25 | 1.1642 | 1.0000 | 1.2177 | 1.0459 | 1.0460 | 0.8984 | 0.8727 | 1.3341 |
| 26 | 0.8514 | 1.0000 | 1.0719 | 1.2591 | 1.2591 | 1.4789 | 1.0000 | 0.8514 |
| 27 | 0.7115 | 1.0001 | 0.9508 | 1.3364 | 1.3363 | 1.8783 | 1.1977 | 0.5940 |
| 28 | 0.6865 | 1.0000 | 1.0115 | 1.4734 | 1.4734 | 2.1461 | 0.9424 | 0.7285 |
| 29 | 0.8815 | 0.9940 | 1.2117 | 1.3664 | 1.3746 | 1.5502 | 1.2159 | 0.7293 |
| 30 | 0.7619 | 1.0000 | 1.0870 | 1.4267 | 1.4266 | 1.8724 | 1.0529 | 0.7236 |
| 31 | 0.7588 | 0.9906 | 1.0740 | 1.4021 | 1.4153 | 1.8478 | 1.0043 | 0.7627 |
| 32 | 0.8135 | 1.0001 | 1.0751 | 1.3217 | 1.3216 | 1.6248 | 0.9094 | 0.8944 |
| 33 | 0.7257 | 1.0040 | 0.9881 | 1.3670 | 1.3616 | 1.8837 | 0.9579 | 0.7546 |
| 34 | 0.9000 | 0.9999 | 1.2015 | 1.3348 | 1.3349 | 1.4830 | 1.1403 | 0.7894 |
| 35 | 0.8480 | 1.0001 | 1.2610 | 1.4871 | 1.4871 | 1.7538 | 1.1587 | 0.7318 |
| 36 | 0.6548 | 0.9999 | 1.0551 | 1.6111 | 1.6112 | 2.4602 | 1.0796 | 0.6066 |
| 37 | 0.8215 | 1.0245 | 1.1059 | 1.3792 | 1.3463 | 1.6789 | 0.9582 | 0.8368 |
| 38 | 0.8621 | 0.9999 | 1.1176 | 1.2963 | 1.2965 | 1.5038 | 1.0662 | 0.8086 |
| 39 | 0.8714 | 1.0000 | 1.2579 | 1.4436 | 1.4436 | 1.6567 | 1.4705 | 0.5926 |
| 40 | 0.5207 | 1.0001 | 0.5837 | 1.1210 | 1.1210 | 2.1528 | 0.6145 | 0.8473 |
| G-Mean | 0.8470 | 1.0006 | 1.1028 | 1.3028 | 1.3019 | 1.5380 | 0.9912 | 0.8540 |
| SD | 0.1261 | 0.0282 | 0.1511 | 0.1207 | 0.1309 | 0.3193 | 0.1570 | 0.1666 |


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