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A robust scenario- model for locating emergency medical services bases: A case study for Ahvaz city in Iran

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Abstract

EMS (Emergency Medical Services) is responsible for pre-hospital care, playing a prominent role in saving lives from death as well as serious damage to their health. In view of the threat of disruptions to the network components and the risk of parameter uncertainties in the real world, it is incumbent upon render expedient EMS systems. In this regard, this paper unveils a two-phase approach based on data envelopment analyses and robust scenario-based mathematical model to design EMS network in an uncertain environment. The first phase applies a data envelopment analysis (DEA) to determine more valid and practical points for candidate locations. In the second phase, the strategic and tactical decisions of the concerned EMS are determined. Inasmuch as the marginal demand areas and patients with emergencies, the concerned model takes into account the location of air ambulance bases in such a way that for transferring patients by air ambulance to hospitals, hospitals are equipped with helipads. The unbiased considerations are also addressed by minimizing the transfer time to the farthest demand areas. Likewise, in a bid to better allocate emergency facilities to patients and the patients to appropriate hospitals with their physical condition, categorizing the type of disease for patients is carried out. Lastly, a real case study of the EMS system of Ahvaz city in Iran is exploited, via which outstanding managerial insights are attained.

Keywords: Emergency Medical Services (EMS) network design, robust optimization, transfer points location

1- Introduction

EMS is substantial component of health systems and responsible for the pre-hospital services such as providing medical services at the demand points and transporting patients to the proper hospitals. Emergency facilities should be present at demand points at the earliest time right after receiving an emergency reports. Medical first aid administrated to patients at the scenes and the ones who need further assistance transported to appropriate medical centers (Aringhieri et al. 2017 and Belanger et al. 2019). Ameliorating response time enhances the quality and efficiency of EMS system. To improve the process the time between receiving an emergency call and arriving at the scene must be shorten and in the events of transferring to the hospital, transfer to the appropriate time (Boujemaa et al. 2018). Although many

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studies have been done in EMS, but according to surveys, the number of papers in their network design to detail is essential to improve system performance, attention has been paid; not much.

Helicopter Emergency Medical Services (HEMS) are a pivotal part of EMS. HEMS help drastically to better assist low-populated and marginal geographic areas. The performance of EMS systems will enhance when the two parts of the ground and air bases integrate simultaneously and eventuate in improve response time effectively. As EMS systems expects to be able to provide the best service in the shortest possible time, meeting this expectation is difficult, given the limited number of facilities and budget constraints. Therefore, proper location of ground and air bases plays an important role in reducing response time (Shahriari et al. 2017 and Li et al. 2011). In this regard, Furuta and Tanaka (2013) formulated a mathematical model of mini-sum and mini-max location problem for HEMS, transfer points, and applied a case study in Japan to validate their model. Bozorgi-Amiri et al. presented a non-linear numerical planning model for the location of the emergency air base and transmission point, with an uncertainty in terms of demand points. This paper assumed that there are three types of transfers. The objective function of this model is to minimize the time of transfer from demand points to hospitals. Due to the fact that helicopters can only land and take off in specific places, locating these areas, which refer to transfer points in this study, is an urgent decision.

Due to the direct impact of response time on patients' health quality, the location of HEMS bases and transfer points for the purpose of transferring patients via these transfer points is a serious challenge for EMS network design. Hosseinijou and Bashiri (2012) have provided a mathematical model for locating transfer points, which is used to get closer to the real world instead of considering the place of demand as a point in the region. Coordinates of demand areas are a function of uniform distribution. Since the importance of all demand points is not the same, Kalantari et al. (2014) have presented a mathematical model for the problem of location of transfer points with weighted demand points to regarding this problem and the coordinates of these points are considered fuzzy. Tofighi et al. (2016) designed a pre-crisis and post-crisis relief network for the city of Tehran with central warehouses and local distribution centers. In their paper, they have developed a novel two-stage scenario-based possibilistic-stochastic programming approach. Since many hospitals are worn out and not equipped with helipads hospital grounds, hospitals can be utilized as candidate sites for helipad construction. A critical element for designing EMS network is to take into account the condition of disruption, being very common in a real word. In view of the fact that disruption of systems is possible, and, system efficiency is significantly reduced in this situation, it is incumbent upon to address the possibility of disruption (Zarrinpoor et al. 2018).

For this purpose, Mohammadi and Yaghoubi (2017) proposed a two-objective mathematical model for locating transfer points and distribution centers. The model addressed the failure probability of routes and disruption of medical supply distribution centers in accordance with disasters. Likewise, it is possible that transfer points are disrupted because of the impossibility of take-off and landing of helicopters at the transfer points for various reasons. Fixing this problem can cause improvement in the effectiveness of the EMS network and it is still a very limited amount of literature that notices the disruption in regards to the EMS network design problem.

For the purpose of better allocation of emergency facilities to patients and the patients to appropriate hospitals with their physical condition, categorizing the severity of their conditions as emergency or non-emergency is another problem. Mohammadi and Yaghoubi (2017) and Boujemaa et al. (2018) taken into account the categorization of patients in their mathematical model formulation. Nonetheless, the body of the literature addressing categorizing the severity of patients' conditions in EMS problem is still very limited.

Another important point is that in a bid to better service emergency facilities to patients, considering objective function base on fairness can be useful. In this sense, Sabouhi et al. (2019) formulated a multi-objective mathematical model with uncertainties for locating shelters and transfer points. The objective function aimed to minimize the maximum transportation time to the purpose of fairness. Their paper deals with both the transfer of injured persons to hospitals as well as the evacuation of affected areas to shelters.

In as much as few studies have been published on formulating objective function based on fairness in EMS network design, further attention to this problem is needed.
In practice, many parameters are hemmed in by a high degree of uncertainties. Overlooking these uncertainties can contribute to poor quality of decisions and infeasible responses (Dehghani et al. 2020). Meanwhile, in relation to health, many parameters such as demand and transfer times are associated with significant changes . On the other side, in view of the fact that it is directly related to people's health, a system must be set up that can meet the current situation. In the field of EMS, many studies, such as Nickel et al. (2016) and Mohammadi et al. (2016), have considered the stochastic approach to capture uncertainty. Furthermore, there are scanty modeling efforts, addressing EMS network design with robust programing. Zhang and Jiang (2014) formulated a multi-objective mathematical model with the aid of the robust programming. Their model simultaneously locates emergency medical service stations and allocates these services to demanded areas as well as determining the number of vehicles in each station. Liu et al. (2019) proposed a robust mathematical model for optimizing the location of ambulance bases and determining the number of facilities per base.
Given the above-mentioned discussion, the main contributions, differing this paper from the relevant literature, is delineated as follows:

- Inasmuch as removing the inappropriate points of the candidate locations can help reduce the size of the problem and leads to reduce the complexity of the mathematical model in this study we applied data envelopment analyses (DEA) to determine more valid and practical transfer points for candidate locations.
- Taking into account hospitals as candidate sites for helipad construction. This is important because many hospitals are worn out and are not equipped with helipads to the purpose of landing and taking off air ambulances.
- Proposing a robust scenario-based mathematical model to design EMS network in an uncertain environment.
- Addressing justice considerations in the objective function. Speaking intuitively, the objective function aims at minimizing the transfer time from ambulance bases to demand points for the farthest demand point as well as demand points to hospital.
- Categorizing emergency facilities and expedient hospitals to patients with their related illnesses.
- Since the fact that disruption of the systems is a possibility, the probability of disruption at transfer points is addressed.
- Validating the concerned model in a case study of EMS via which outstanding managerial insights are gained.

This paper is organized as follows. The second section of this paper includes the description of the concerned EMS problem and DEA method. Section 3, presents the mathematical model of the problem along with applying a robust scenario-based approach is presented. Section 4 presents the case study and incudes computational results and discussions. Finally, Section 5 concludes conclusion and further research.

2- Problem definition

To design an efficient EMS network taking into account the complexities of the real world, it is essential to consider diverse issues including the location of air ambulance bases as well as the transfer points for patients with emergencies to save time. Attention should be paid to the uncertainty of many parameters, taking into account the categories of patients and appropriately-equipped hospitals and the need for transferring patients to medical centers or providing on-scene first aid, as well as paying attention to the risk of disruption in the transfer points in such situations and using unbiased considerations to determine the objective function.

In this section, according to what is happening in the real case study, we put forward a mathematical model for the design of an EMS network. The proposed model deals with positioning locating ground and air ambulance bases, transfer points and hospital selection for helipad construction as well as patient flow

between these points. The allocation of ground and air ambulances between bases is important given the limited number of these facilities. The optimal number of ambulances and air ambulances for each base is determined by the model depending on the quantity of services provided by each base. The uncertainty in this problem is based on the demand parameter (the number of patients in each of the demand points depending on the type of disease) and the time (including the transition time between points). The modeling method in this problem is integer linear programming. Some variables are binary and some of them are integer variables. Figure1 illustrates the schema of our EMS network model in this paper.

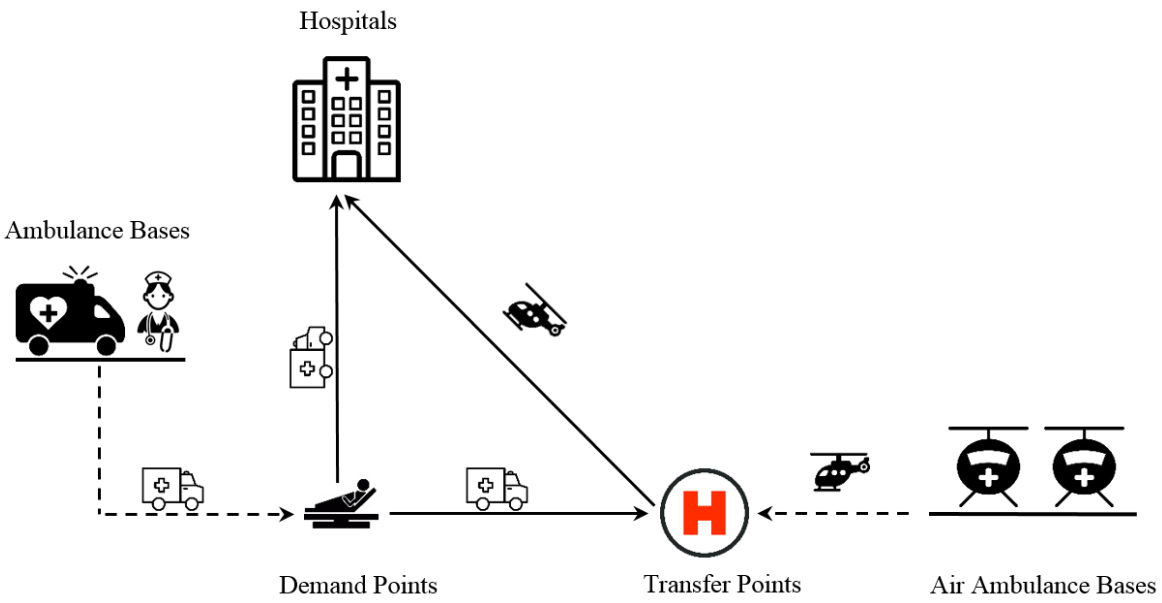


Fig 1. General schema of our EMS network

After emergency facilities arrive to demand points and provide medical aid, if it is determined that the patient needs to be transferred to the hospital, it is important to consider the capacity and expertise of the hospital as outlined in the model presented here. Because of the special health condition of some patients, it is not possible to transport these patients by ground ambulance to hospitals, and high-speed facilities such as air ambulances are used to save time. In this situation, ambulances transport patients to the transfer points, which is an appropriate place for the air ambulances to land and take off. During this time, an air ambulance moves from the base to the transfer point, and after the ambulance and the air ambulance arrive at the transfer point, the air ambulance transfers the patient to the appropriate hospital. At the proposed network, there are two ways to transfer patients to the hospital: **1.** the patient is transported directly to the hospital by ambulance from the request point. **2.** The patient is transported to a helipad-equipped hospital by the air ambulance via the transfer point. Due to the fact that disruption of the systems is a possibility, in many cases, it is not possible for air ambulances to take off or land at the transfer points. Under these circumstances, the transfer point is disrupted. Since many hospitals are outdated and are not equipped with a helipad to assist air ambulances in landing and taking off on the hospital grounds, we can locate hospitals as candidate sites for helipads. In this case, the patients who are transferred to the transfer points will be transferred only to the hospitals equipped with a helipad.

The target network deals with the positioning of ambulance bases, air ambulance bases and transfer points. Patient allocation is also determined between the different components of the network and the optimal number of ambulances assigned to each base. The objective function of the proposed mathematical model is to minimize the transportation time based on fairness with regard to rural and urban patients.

2-1- DEA model

Data envelopment analysis is a non-parametric method for evaluating decision-making units (DMUs) based on observations. This technique manages complex relationships between inputs and outputs and does not require pre-defined weights and normalization of input and output units (Dehghani et al. 2018) .
Farrell (1957) conducted preliminary studies in DEA. In this study, performance measurements included an input and an output. Charnes, Cooper and Rhodes (1978), developed Farrell's view and provided a model that was able to measure performance with multiple inputs and outputs. This model is known in the literature as the CCR model and considers the constant return to scale. Banker, Charnes, and Cooper (1984), considered a model with a variable return to scale that is known in the literature as the BCC model.

The main concept of classical DEA is to identify the production frontier where the DMUs are efficient and the score of the units that are not on the efficient frontier is obtained by comparing them with the efficient DMUs. It is worth note that all units on the efficient frontier have the same score and the highest score. In fact, many DMUs are on the efficient frontier, which makes the mentioned units indistinguishable and makes the performance of DEA ranking difficult.
In this study, the method of Shen et al. (2016) was used to evaluate different locations for transfer points. In the proposed method, an index is used that simultaneously uses the distance between the efficient and anti-efficient frontiers, also standard cover analysis and reverse cover analysis are used simultaneously to provide more information about frontiers. This approach will provide a better ranking. The approach used is described below.

Assume that there are n number of DMUs so that their index is represented by $D(d = 1, \dots, n)$. Also, the inputs and outputs of the data envelopment analysis model for DMUs are $x_{ed}(e = 1, \dots, i)$ and $y_{fe}(f = 1, \dots, j)$, respectively. The following model shows the standard DEA model:

$$min\, h_{bk} = \omega_k \tag{1}$$

$$\sum_{d=1}^{n} x_{ed}\mu_d \leq \omega_k x_{ek} \quad e = 1, \dots, i \tag{2}$$

$$\sum_{d=1}^{n} y_{fe}\mu_d \geq y_{fk} \quad f = 1, \dots, j \tag{3}$$

$$\mu_d \geq 0, \qquad d = 1, \dots, n$$
$$\omega_k\ unconstrained. \tag{4}$$

Furthermore, the inverted DEA model is as follows:

$$max\, h_{wk} = \omega_k \tag{5}$$

$$\sum_{d=1}^{n} x_{ed}\mu_d \geq \omega_k x_{ek} \quad e = 1, \dots, i \tag{6}$$

$$\sum_{d=1}^{n} y_{fe}\mu_d \leq y_{fk} \quad f = 1, \dots, j \tag{7}$$

$$\mu_d \geq 0, \qquad d = 1, \dots, n$$
$$\omega_k\ unconstrained. \tag{8}$$

In the proposed model, ω_k is the efficiency measure of DMU "k", x_{ek} and y_{fk} show the inputs and outputs for DMU "k", respectively and μ_d is the dual weight attributed to all inputs and outputs of DMU "d".

The models are solved n times as efficient and anti-efficient frontiers of standard and inverted DEA models. To exploit information of both efficient and anti-efficient frontiers, for each DMU the following formulate is calculated.

$$hl_k = \frac{\left[h_{bk}^* + \left(1 - \frac{1}{h_{wk}^*}\right)\right]}{2} \tag{9}$$

A set of geographical, managerial and technical criteria has been considered to evaluate the location of transfer points. In the following, we will introduce and explain the considered criteria.

- **Distance from the hospital**

The appropriate distance between transfer points and hospitals will improve the transfer time. Therefore, distance from the hospital is considered as an input parameter.

- **Disaster**

Due to the reasons that disruption of transfer point in many cases it is not possible for air ambulances to take off or land at the transfer points. According to this criteria is essential and therefore, this parameter is considered as an input parameter.

- **Population density**

Selecting a transfer point in areas with higher population densities can be the cause of better services. Hence, this parameter is considered as an output parameter.

- **Availability**

Selecting a high access point is strength and can lead to better services therefore availability is an output parameter.

3- Model formulation

Before formulating the problem, and carrying out an observation of the encyclopedic illustration of the proposed EMS network design model, the following verbal description of it is given:

Minimization of maximum transfer time *= transportation time from ambulance bases to demand points + transportation time from demand points to hospitals*

Subject to:

Demand satisfaction equations
Network balance equations
Capacity equations
The budget equation
The number of emergency facilities per base
Binary and integer decision variables

3- 1- Parameters and decision variables

The indices, parameters and decision variables in the problem will be introduced in the following:

I	Set of demand points, indexed by *i*
M	Set of ambulance bases, indexed by *m*
K	Set of air ambulance bases, indexed by *k*
J	Set of transfer points, indexed by *j*
H	Set of hospitals, indexed by *h*
L	Set of patients categories, indexed by *l*
S	Set of possible scenarios, indexed by *s*

Parameters:

tad_{mi}^{s}	Transportation time between ambulance base *m* and demand point *i* under scenario *s*
tdh_{ih}^{s}	Transportation time between demand point *i* and hospital *h* under scenario *s*
tdt_{ij}^{s}	Transportation time between demand point *i* and transfer point *j* under scenario *s*
tst_{kj}^{s}	Transportation time between air ambulance base k and transfer point *j* under scenario *s*
tth_{jh}^{s}	Transportation time between transfer point *j* and hospital *h* under scenario *s*
$e = max\{tdt_{ij}^{s}, tst_{kj}^{s}\}$	$\forall i \in I, \forall j \in J, \forall\, k \in K$
p_{l}^{s}	The probability of transferring to the hospital for the patient category *l* under scenario *s*
w_{l}	Weight of the patient category *l*
n_{il}^{s}	The number of patients with category *l* at demand point *i* under scenario *s*
$capa_{m}$	Capacity for ambulance base at location *m*
$capt_{j}$	Capacity for transfer point at location *j*
$caps_{k}$	Capacity for air ambulance base at location *k*
$caph_{h}$	Capacity for hospital at location *h*
qa	The capacity of each ambulance base during a specific period of time
qs	The capacity of each ambulance base during a specific period of time
ab_{hl}	1 if hospital h can treat patient category *l*, 0 otherwise.
na	The total number of ambulances
ns	The total number of air ambulances
b	The total available budget
ca_{m}	Fixed cost of locating an ambulance base at location *m*
cs_{k}	Fixed cost of locating an air ambulance base at location *k*
ct_{j}	Fixed cost of locating a transfer point at location *j*
cst_{kj}	Transportation cost for a patient from air ambulance base *k* to transfer point *j* with air ambulance
cth_{jh}	Transportation cost for a patient from transfer point *j* to hospital *h* with air ambulance
ψ_{j}^{s}	1 if transfer point *j* be disrupted under scenario *s* is located, 0 otherwise.

Decision variables:

xa_{m}	1 if ambulance base *m* is located, 0 otherwise.
xs_{k}	1 if air ambulance base *k* is located, 0 otherwise.
xt_{j}	1 if transfer point *j* is located, 0 otherwise.
xh_{h}	1 if hospital *h* is equipped with helipad, 0 otherwise.
yad_{mil}^{s}	The number of patients with category *l* who should be allocated from demand point *i* to ambulance base m under scenario *s*
yd_{mil}^{s}	The number of patients with category *l* treated on demand point *i* by ambulance base *m* under scenario *s*
ydh_{mihl}^{s}	The number of patients with category *l* who should be transferred from demand point *i* to hospital h by ambulances of ambulance base m under scenario *s*

ydt^s_{mijl}	The number of patients with category *l* who should be transferred from demand point *i* to transfer point *j* by ambulances of ambulance base *m* under scenario *s*
yst^s_{kjl}	The number of patients with category *l* who should be allocated from transfer point *j* to air ambulance base *k* under scenario *s*
yth^s_{kjhl}	The number of patients with category *l* who should be transferred from transfer point *j* to hospital *h* with air ambulances of air ambulance base *k* under scenario *s*
za_m	Number of ambulances assigned to the ambulance base *m*
zs_k	Number of air ambulances assigned to the air ambulance base *k*

3-2- Objective function

Since the proposed model is in the field of health care and is designed for the emergency medical service system, this section pertains to emergency issues. Therefore, minimizing the maximum time for patients to be transferred to hospitals is crucial for all patients that is, both patients those who are available in the area and as well as those patients who are in remote and marginal areas. The objective function aims at minimizing the transfer time. Likewise, the unbiased considerations are applied to minimize the transfer time to the farthest demand areas .The mentioned objective function comprises of two components, which is formulated as follows:

$$\min Z = \max_{i\in I}\left(\sum_{s}\pi^s\left(\sum_{m\in M}\sum_{l\in L} w_l tad^s_{mi} yad^s_{mil}\right)\right) + \max_{i\in I}\left(\sum_{s\in S}\pi^s\left(\sum_{m\in M}\sum_{h\in H}\sum_{l\in L} w_l tdh^s_{ih} ydh^s_{mihl}\right), \sum_{s\in S}\pi^s\left(\sum_{l\in L} w_l\left(\sum_{m\in M} e \cdot ydt^s_{mijl} + \sum_{k\in K}\sum_{j\in J} tth^s_{jh} yth^s_{kjhl}\right)\right)\right) \tag{10}$$

The objective function minimizes the demand-weighted transfer time from ambulance bases to demand points for the farthest demand point as well as demand points to hospitals. Since there are two possible ways for transporting patients from demand points to hospitals (direct transportation by ambulance to transfer points and transporting by air ambulances to hospitals), the second part of the objective function includes two terms.

3-3- Model constraints

In this section, the constraints of the mathematical model and their introduction are discussed.

$$\sum_{m\in M}\sum_{h\in H} ydh^s_{mihl} + \sum_{m\in M}\sum_{j\in J} ydt^s_{mijl} = p^s_l n^s_{il} \qquad \forall i\in I, \forall l\in L, \forall s\in S \tag{11}$$

$$\sum_{m\in M} yd_{mil}^{s} = (1-p_l^s)n_{il}^s \qquad \forall i\in I, \forall l\in L, \forall s\in S \tag{12}$$

$$yad_{mil}^{s} = \sum_{h\in H} ydh_{mihl}^{s} + \sum_{j\in J} ydt_{mijl}^{s} + yd_{mil}^{s} \qquad \forall m\in M, \forall i\in I, \forall l\in L, \forall s\in S \tag{13}$$

$$\sum_{m\in M}\sum_{i\in I} ydt_{mijl}^{s} = \sum_{k\in K}\sum_{h\in H} yth_{kjhl}^{s} \qquad \forall j\in J, \forall l\in L, \forall s\in S \tag{14}$$

$$yst_{kjl}^{s} = \sum_{h\in H} yth_{kjhl}^{s} \qquad \forall k\in K, \forall j\in J, \forall l\in L, \forall s\in S \tag{15}$$

$$\sum_{i\in I}\sum_{l\in L} yad_{mil}^{s} \le capa_m xa_m \qquad \forall m\in M, \forall s\in S \tag{16}$$

$$\sum_{m\in M}\sum_{i\in I}\sum_{l\in L} ydt_{mijl}^{s} \le capt_j(1-\psi_j^s)xt_j \qquad \forall j\in J, \forall s\in S \tag{17}$$

$$\sum_{j\in J}\sum_{l\in L} yst_{kjl}^{s} \le caps_k xs_k \qquad \forall k\in K, \forall s\in S \tag{18}$$

$$\sum_{m\in M}\sum_{i\in I} ydh_{mihl}^{s} + \sum_{k\in K}\sum_{j\in J} yth_{kjhl}^{s} \le caph_h ab_{hl} \qquad \forall h\in H, \forall l\in L, \forall s\in S \tag{19}$$

$$\sum_{k\in K}\sum_{j\in J}\sum_{l\in L} yth_{kjhl}^{s} \le caph_h xh_h \qquad \forall h\in H, \forall s\in S \tag{20}$$

$$\sum_{m\in M} ca_m xa_m + \sum_{k\in K} cs_k xs_k + \sum_{j\in J} ct_j xt_j + \sum_{h\in H} ch_h xh_h + \sum_{k\in K}\sum_{j\in J}\sum_{h\in H}\sum_{l\in L}(cst_{kj}+cth_{jh})yth_{kjhl}^{s} \le B \tag{21}$$

$$za_m \le M\cdot xa_m \qquad \forall m\in M \tag{22}$$

$$\sum_{m\in M} za_m \le na \tag{23}$$

$$\sum_{i\in I}\sum_{l\in L} yad_{mil}^{s} \le za_m qa \qquad \forall m\in M, \forall s\in S \tag{24}$$

$$\sum_{k\in K} zs_k \le ns \tag{25}$$

$$zs_k \le M\cdot xs_k \qquad k\in K \tag{26}$$

$$\sum_{j\in J}\sum_{l\in L} yst_{kjl}^{s} \le zs_k qs \qquad k\in K, \forall s\in S \tag{27}$$

$$xa_m, xt_j, xs_k, xh_h \in \{0.1\} \tag{28}$$

$$yad_{mil}^{s}, yd_{mil}^{s}, ydt_{mijl}^{s}, yst_{kjl}^{s}, yth_{kjhl}^{s}, za_m, zs_k \ge 0. Integer \tag{29}$$

Equations (11) and (12) ensure that the demands should be satisfied, so equation (11) applies to patients who need to transfer to hospital. Equation (12) applies to patients who receive treatment at the scene.

Equation (13) shows that each demand point assigned to ambulance bases either is treated at the scene by the ambulances team or should be transferred to hospitals directly or via transfer points. Equations (14) and (15) are transfer points balance equations and control the input and output flows from these points. Equations (16) to (20) are capacity constraints of ambulance bases, transfer points pondering the probability of disruption at transfer points, air ambulance bases and hospitals. Equation (21) specifies the budget constraint. Equations (22) to (27) demonstrate the allocation of ground and air ambulances between bases and indicate the number of emergency facilities for every ground and air base that can be activated. Equations (28) and (29) present the type of the variables.

3-3-1- Linearization of the model

The non-linearity problem of the objective function is a serious problem, which the model proposed in the previous section is entangled with. Accordingly, in this section, some linearization methods are exploited to capture the aforementioned problem. The objective function of the mathematical model presented in equation (10) is of the minimum maximization type and therefore this equation is nonlinear. To linearize the objective function, T and T' variables are defined and the objective function is written as equation (30). Furthermore, equations of (31) to (34) are also added to the model.

$$\min Z = T + T' \tag{30}$$

$$T \geq \sum_{s\in S} \pi^s \left(\sum_{m\in M} \sum_{l\in L} w_l tad^s_{mi} yad^s_{mil} \right) \qquad \forall i \in I \tag{31}$$

$$T' \geq \sum_{s\in S} \pi^s \left(\sum_{m\in M} \sum_{h\in H} \sum_{l\in L} w_l tdh^s_{ih} ydh^s_{mihl} \right) \qquad \forall i \in I \tag{32}$$

$$T' \geq \sum_{s\in S} \pi^s \left(\sum_{l\in L} w_l \left(\sum_{m\in M} e \cdot ydt^s_{mijl} + \sum_{k\in K} \sum_{j\in J} \sum_{h\in H} tth^s_{jh} yth^s_{kjhl} \right) \right) \qquad \forall i \in I \tag{33}$$

$$T, T' \geq 0 \tag{34}$$

In the equation (30), T is replaced with the first item of objective function, (minimizing the demand-weighted transfer time from ambulance bases to demand points for the farthest demand point) as well as T' for the second item of objective function (demand points to hospitals). Equation (31) demonstrates the maximum transfer time for arriving ambulances from ambulances bases to demand points. Equations (32) and (33) set the maximum transfer time for transporting patients from demand points to hospitals (directly by ambulances to hospitals or via transfer point through transporting by air ambulances to hospitals). Equation (34) presents the type of the new variables.

3-3-2- Robust scenario-based stochastic programming method

As mentioned before, some parameters of the proposed optimization model are hemmed in by uncertainties. Regarding to the scenario base approach of our network design model, and uncertain parameters, in this paper we have selected the robust approach introduced by Mulvey et al. (1995) to deal with uncertainty.

In the robust approach introduced by Mulvey et al. (1995) comprise two sets of robustness: (1) solution robustness and (2) model robustness. The first set deals with seeking a near optimal solution for all scenarios and the second set deals with applying an approximately feasible solution for all scenarios by pondering penalty function in objective function. In the following, we concisely expound the robust scenario-based stochastic formulation proposed by Mulvey et al. (1995).

$$\min c^T x + d^T y \qquad x \in R^{n_1}, y \in R^{n_2} \tag{35}$$

$$Ax = b \tag{36}$$

$$Bx + Cy = e \tag{37}$$

$$x.y \geq 0 \tag{38}$$

Where, $x \in R^{n_1}$ be a vector of design variables and $y \in R^{n_2}$ be a vector of control variables. Equation (36) is the fixed constraint, which is free of noise, equation (37), is the control constraint, which is subject to noise.

Equation (36) is the structural constraint and equation (37) is the control constraint, and the coefficients of equation (36) are fixed and free of noise whilst coefficients of equation (37) are subject to noise.

The probability of each scenario is equal to π^s so that $\sum_{s \in S} \pi^s = 1$. The robust scenario-based optimization by Mulvey et al. (1995) in general is as follows:

$$\min \sigma(x.y_1.y_2.\cdots.y_s) + \mu\rho(v_1.v_2.\cdots.v_s) \tag{39}$$

s.t.

$$Ax = b \tag{40}$$

$$B_s x + C_s y_s + u_s = e_s \qquad \forall s \in S \tag{41}$$

$$x \geq 0. y_s \geq 0 \qquad \forall s \in S \tag{42}$$

Due to the uncertain parameters, some equations may be unmet under some scenarios. For this reason the control variable v_s, which represents the infeasibility of the model under scenario *s*, must be determined. Equation (39) shows the objective function, which includes two parts. The first part represents the solution robustness, and the second part is the model robustness weighted by μ. The general form suggested for the first part in the Mulvey et al. (1995) is as follows:

$$\sigma(x.y_1.y_2.\cdots.y_s) = \sum_{s \in S} \pi^s \xi_s + \lambda \sum_{s \in S} \pi^s \left(\xi_s - \sum_{s' \in S} \pi^{s'} \xi_{s'} \right)^2 \tag{43}$$

Equation (43) shows, in addition to regarding the objective function, the solution variance is regarded. Due to the quadratic term in second part of formulation of equation (43), Yu and Li (2000) presented equation (44) with absolute deviation term instead of the quadratic term in equation (43).

$$\sigma(x.y_1.y_2.\cdots.y_s) = \sum_{s \in S} \pi^s \xi_s + \lambda \sum_{s \in S} \pi^s \left| \xi_s - \sum_{s' \in S} \pi^{s'} \xi_{s'} \right| \tag{44}$$

Leung et al. (2007) proposed linear formulation to solve the absolute term of formulation proposed by Yu and Li (2000) as follows:

$$\min z = \sum_{s \in S} \pi^s \xi_s + \lambda \sum_{s \in S} \pi^s \left[\left(\xi_s - \sum_{s' \in S} \pi^{s'} \xi_{s'} \right) + 2u_s \right] \tag{45}$$

s.t.

$$\xi_s - \sum_{s' \in S} \pi^{s'} \xi_{s'} + u_s \geq 0 \qquad \forall s \in S \tag{46}$$

$$u_s \geq 0 \qquad \forall s \in S \tag{47}$$

It should be noted that in cases where $\xi_s$is greater than $\sum_{s'\in S}\pi^{s'}\xi_{s'}$, then $u_s = 0$ and, whenever $\sum_{s'\in S}\pi^{s'}\xi_{s'}$ is greater than ξ_s, then $u_s = \sum_{s'\in S}\pi^{s'}\xi_{s'} - \xi_s$. To express the model robustness, any encroachment of the control constraint, should be penalized. As a result, the robust objective function presented in equation (39), can be formulated as:

$$\min \sum_{s\in S}\pi^s\,\xi_s + \lambda\sum_{s\in S}\pi^s\left[\left(\xi_s - \sum_{s'\in S}\pi^{s'}\,\xi_{s'}\right) + 2u_s\right] + \mu\sum_{s\in S}\pi^s\delta_s \tag{48}$$

To formulate the proposed model to robust approach introduced by Leung et al. (2007), we consider δ_{il}^s as the unmet demand under scenario *s*. In view of the risk of disruptions to the network or under some situations in the real world, which leads to a decrease in the performance of the health systems and as a result leads to serious damages to health or even death, considering the penalty cost μ, is prominent in the objective function.

To convert our proposed model to robust approach introduced by Leung et al. (2007), we have the following formulation:

$$\min Z_s = T_s + T'_s \qquad \forall s \in S \tag{49}$$

$$T_s \geq \sum_{m\in M}\sum_{l\in L} w_l tad_{mi}^s yad_{mil}^s \qquad \forall i \in I, \forall s \in S \tag{50}$$

$$T'_s \geq \left(\sum_{m\in M}\sum_{h\in H}\sum_{l\in L} w_l tdh_{ih}^s ydh_{mihl}^s\right) \qquad \forall i \in I, \forall s \in S \tag{51}$$

$$T'_s \geq \sum_{l\in L} w_l\left(\sum_{m\in M} e\cdot ydt_{mijl}^s + \sum_{k\in K}\sum_{j\in J}\sum_{h\in H} tth_{jh}^s yth_{kjhl}^s\right) \qquad \forall i \in I, \forall s \in S \tag{52}$$

$$T'_s \geq \sum_{l\in L} w_l\left(\sum_{m\in M} f\cdot ydt_{mijl}^s + \sum_{k'\in K'}\sum_{j\in J}\sum_{h\in H} tth_{jh}^s ythb_{k'jhl}^s\right) \qquad \forall i \in I, \forall s \in S \tag{53}$$

$$\sum_{m\in M}\sum_{h\in H} ydh_{mihl}^s + \sum_{m\in M}\sum_{j\in J} ydt_{mijl}^s + \delta_{il}^s = p_l^s n_{il}^s \qquad \forall\in I, \forall l \in L, \forall s \in S \tag{54}$$

$$\sum_{m\in M} yd_{mil}^s + \delta'^s_{il} = (1 - p_l^s)n_{il}^s \qquad \forall\in I, \forall l \in L, \forall s \in S \tag{55}$$

$$z_s - \sum_{s'\in S}\pi^{s'} z_{s'} + u_s \geq 0 \tag{56}$$

It should be noted that $Z_{s'}$ is the same as Z_s, in which s' is used instead of s. Eventually the robust scenario-base programming model is formulated as follows:

$$Minimize \sum_{s\in S} \pi^s Z_s + \lambda \sum_{s\in S} \pi^s \left[\left(Z_s - \sum_{s'\in S} \pi^{s'} Z_{s'}\right) + 2u_s\right] + \mu \sum_{s\in S} \pi^s (\delta_{il}^s + \delta'^s_{il}) \tag{57}$$

$$u_s \geq 0, \delta_{il}^s \geq 0, \delta'^s_{il} \geq 0 \tag{58}$$

4- Case study

According to official statistics released by the Deputy of Planning & Development Ahvaz municipality, out of all the days in the year, in 130 of them the air quality in the city of Ahvaz was in bad condition. This can cause respiratory problems, heart problems and other diseases for many people in this area and increase the demand for emergency medical services. Accordingly, in this paper, the proposed problem is applied in Ahvaz city.

Based on historical data, we applied our model to EMS of Ahvaz city in Iran. Ahvaz city is located in Khuzestan province, a western province of Iran. According to the official statistics released by the Ahvaz municipality, the population of Ahvaz in 2017 is 1.199.953 people. Because of the bad weather on many days of the year in this area, and subsequently created health problems for its citizens, decisions regarding appropriate EMS are very important.

The data used in this case study is provided from reports of Deputy of Planning & Development Ahvaz municipality https://planning.ahvaz.ir/. The collected data includes number of patients in each demanded area, transportation times, and capacity of each ambulance base, air ambulance bases, transfer points and hospitals, and the number of emergency facilities available.

- We divided patients into emergency and non-emergency patients. Because of the greater importance of emergency patients, these patients' weight is 0.7 and normal patients' weight is 0.3 in the objective function. In this study, it is assumed that all emergency patients are transferred to the hospital and that non-emergency patients are being transferred to the hospital with a probability of 0.5 (i.e. half of the non-emergency patients).
- We considered four scenarios, in terms of no traffic with appropriate weather conditions, no traffic with inappropriate weather conditions, heavy traffic with appropriate weather conditions and heavy traffic with inappropriate weather conditions. The reason for these divisions is that the city of Ahvaz is in very unfavorable weather conditions due to air pollution on many days of the year, in which case it is not possible for air ambulances to take off or land at the transfer points. Under these situations, the transfer point is disrupted. Uncertainty in the transfer time is also considered by considering smooth traffic or high traffic.
- For calculating the transportation time between the nodes, we applied the speeds derived from Google Maps, while to calculate the transportation time by air ambulances, we assume an air ambulance speed of 200 km per hour.
- We considered a total of 24 points in Ahvaz as the demand points. In this problem, 21 potential locations for ambulance bases are considered. In addition, to determine these potential locations, we considered the population of the city sections.
- Two potential locations for air ambulance bases are considered. One of them is near the Ahvaz international airport and the other is determined in Padadshahr area. In addition to Ahvaz international airport, the stadiums of Ahvaz city are designated as transfer points, whose names are listed in table.3. Moreover, we intended 13 hospitals in Ahvaz city to transport patients to these medical centers, and the names of these hospitals are shown in table.2.

4-1- Data gathering

In this section, while introducing the case study in more details, the data gathering used in this study is introduced. The city of Ahvaz comprises eight municipal sectors. To determine the demand points, three demand points are considered in each municipal section. The demand of that region is divided into these three mentioned points. Thus despite eight municipal sections, we have considered 24 demand points for

the city of Ahvaz. The population of each section is shown in figure 2. According to experts, the number of Ahvaz EMS missions during a one-month period is about 4200 cases. By obtaining information about the total demand in a month to estimate the demand of each of the demand points, based on the population of municipal sections, the demand of each demand point have been estimated which is shown in table 1. According to the available information, the city of Ahvaz is currently equipped with 17 ambulance bases under the surveillance of Ahvaz Jundishapur University of Medical Sciences. In addition to the existing bases, a number of ambulance bases have been proposed for establishment. The proposed points are such that according to the section of Ahvaz Municipality, at least two ambulance bases in each section would have existed. Thus, a number of ambulance stations have been proposed for establishment according to Table 2. In this table, bases No. 1 to 17 are introduced as existing bases and bases No. 18 to 21 as proposed bases for establishment.

To obtain the probability of occurrence of each considered scenario, according to official statistics released by the Deputy of Planning & Development Ahvaz municipality, considering the high traffic hours in a day and inappropriate weather conditions days in the year the probabilities of the scenarios are as follows: 0.7005, 0.2335, 0.0495 and 0.0615 respectively.

We proposed 15,000,000,000 Rial as a total budget of this plan. We purposed 5,000,000,000 Rial as establishment cost of each ambulance base and 10,000,000,000 Rial as establishment cost of each air ambulance base. The establishment cost of each transfer point is estimated at 1,000,000,000 Rial and the establishment cost of the helipad near the hospitals is estimated at 2,000,000,000 Rial.

We proposed that the service capacity of each ambulance base is equal to providing services to 250 patients during a period, and that each air ambulance base is equal to providing services to 30 patients. The capacity of each transfer point is considered 500 patients during a period. This problem is solved with considering 21 available ambulances and 1 air ambulance.

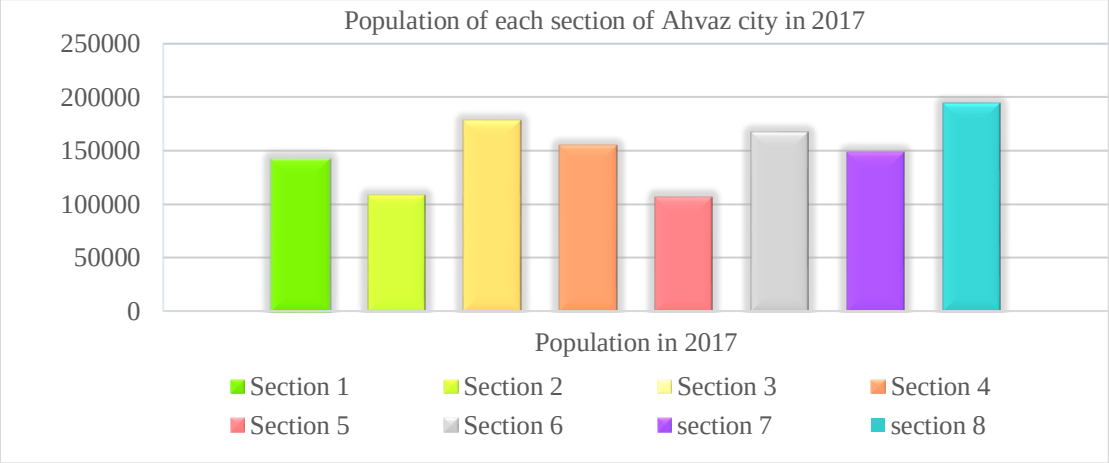


Fig 2. Population of each section of Ahvaz city in 2017

Table 1. Number of patients at demand points under scenarios

	Demand Points	Number of non-emergency patients	Number of non-emergency patients
Number 1	MelliRah Square	(62-62-72-72)	(146-146-156-156)
Number 2	Kuy-e-Farhangiyan	(62-62-72-72)	(146-146-156-156)
Number 3	Shahrak-e Naft	(62-62-72-72)	(146-146-156-156)
Number 4	Zeitoun Kargari	(52-52-62-62)	(122-122-132-132)
Number 5	Karun street	(52-52-62-62)	(122-122-132-132)
Number 6	Sepidar	(52-52-62-62)	(120-120-120-120)
Number 7	Moein Garden	(50-50-60-60)	(116-116-126-126)
Number 8	Sheykh Garden	(50-50-60-60)	(116-116-126-126)
Number 9	Kuy-e-Taleghani	(50-50-60-60)	(116-116-126-126)
Number 10	Kuy-e-Mahdis	(72-72-82-82)	(150-150-160-160)
Number 11	Kuy-e-Isar	(74-74-84-84)	(150-150-160-160)
Number 12	Manazel-e Foolad	(74-74-84-84)	(150-150-160-160)
Number 13	Kian-Abad	(38-38-48-48)	(88-88-98-98)
Number 14	Kianpars	(38-38-48-48)	(88-88-98-98)
Number 15	Amaniyeh	(38-38-48-48)	(88-88-98-98)
Number 16	Manazel-e Rahahan	(58-58-68-68)	(138-138-148-148)
Number 17	Kuy-e-Enghelab	(58-58-68-68)	(138-138-148-148)
Number 18	Sayyahi	(60-60-70-70)	(138-138-148-148)
Number 19	Lashkar-Abad	(54-54-64-64)	(126-126-136-136)
Number 20	Golestan	(54-54-64-64)	(126-126-136-136)
Number 21	Moeinzadeh	(54-54-64-64)	(126-126-136-136)
Number 22	Shahrak-e-Razmandegan	(38-38-48-48)	(88-88-98-98)
Number 23	Pardis	(38-38-48-48)	(88-88-98-98)
Number 24	Shahrak-e-Payam	(38-38-48-48)	(88-88-98-98)

Table 2. List of potential points for transfer points, ambulance bases and hospitals

Hospital		Ambulance Bases	
Number 1	Naft Grand Hospital	Number 1	Chahar Shir
Number 2	Amir Al Momenin Hospital	Number 2	Kuy-e- Mellat
Number 3	Ahvaz Imam Khomeini Hospital	Number 3	Ahvaz International Airport
Number 4	Ahvaz Fatemeh Zahra Hospital	Number 4	Sadeghiyeh
Number 5	Taleghani Hospital	Number 5	17-Shahrivar
Number 6	Razi Hospital	Number 6	Rahband
Number 7	Milad Hospital	Number 7	Sepidar
Number 8	Golestan Hospital	Number 8	Marashi
Number 9	Allameh Karami Hospital	Number 9	Padad Phase two
Number 10	Aboozar Children's Hospital	Number 10	Kianpars
Number 11	Sina Hospital	Number 11	Kian-Abad
Number 12	Niaki martyr army Hospital	Number 12	Kompelo
Number 13	Shahid Baghaei Hospital	Number 13	Chamran
		Number 14	Golestan
		Number 15	17-Ganeh Terminal
		Number 16	Baharestan
		Number 17	Pardis
		Number 18	Foolad Arena Sport Complex
		Number 19	Petroleum University of Technology
		Number 20	Shahid Sobhani Park
		Number 21	Golzar Park

4-2- DEA model implementation

In this section, candidate locations for transfer points are introduced. In point of fact, the transfer points that get the least satisfaction score from a managerial point of view are selected as the candidate locations. The candidate locations obtained in this section are used for optimization in mathematical models.

The proposed algorithm for obtaining the indicators presented in section 5 is encoded in GAMS/ Cplex on an Intel(R) Core (TM) i5-4210U computer with 6GB RAM under Win 8.1.

The minimum score required for the hl_kindex, according to management views, is 0.5. Thus, from the 10 proposed locations, 6 points whose index value hl_k is greater than 0.5 are selected as candidate locations. The results of this section are shown in table 3. As can be seen in table 3, candidate locations 1 to 6, whose scores in the index above are greater than 0.5, are selected as candidate points for transfer points.

Table 3. Efficiency and anti- efficiency results and ranks of transfer points

DMU	Transfer Point	h_{bk}	h_{wk}	hl_k	Rank
DUM 1	Ahvaz International Airport	1.000	10.000	0.950	2
DUM 2	Foolad Arena Sport complex	0.480	6.400	0.662	4
DUM 3	Takhti Sport complex	1.000	13.333	0.962	1
DUM 4	Ghadir Sport complex	0.439	5.714	0.632	6
DUM 5	Azadi Sport complex	0.45	6.000	0.642	5
DUM 6	Mokhaberat Stadium	0.600	8.000	0.737	3
DUM 7	Chamran University Stadium	0.075	1.000	0.038	10
DUM 8	Shahrak-e Naft Stadium	0.150	1.611	0.265	7
DUM 9	Farahani Stadium	0.129	1.000	0.064	9
DUM 10	Phase two Stadium	0.133	1.000	0.100	8

It is worth note that removing the inappropriate by applying the proposed approach can eventuate in reducing the complexity of the model. Likewise, the most applicable locations are selected as candidate points for transfer points.

4-3- Computational results

This section is intended to explain and discuss the numerical results related to studied case. In doing so, the concerned model is solved by GAMS/Cplex on an Intel(R) Core (TM) i5-4210U computer with 6GB RAM under Win 8.1. The computation time for running took about twenty minutes on average.

The numbers of activated facilities (ambulance bases, air ambulance bases and transfer points) have been demonstrated in the table 4 and figure 3. Table 4 shows that after solving the proposed model, bases No. 1 to 17, which were previously active in the city of Ahvaz, are re-selected to provide services, and in addition, bases No. 18 and 19 are located Proposals for the establishment of ambulance bases have been activated.

Table 4. Results of current and optimal locations

Current Location Of Ambulance bases	Optimal Location Of Ambulance bases	Current Location Of Air Ambulance bases	Optimal Location Of Air Ambulance bases	Optimal Location Of Transfer Point
1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17	1-2-3-4-5-6-7-8-9-10-11-12-13-14-15-16-17-18-19	1	1	1-3

The numbers of areas, which are assigned to activate facilities, are showed in the Table 5. The following table shows what demand points are assigned to each activated ambulance base as well as activated transfer points and hospitals. Due to the multiplicity of flow variables identifiers, it is sufficient to report the results for the first scenario.

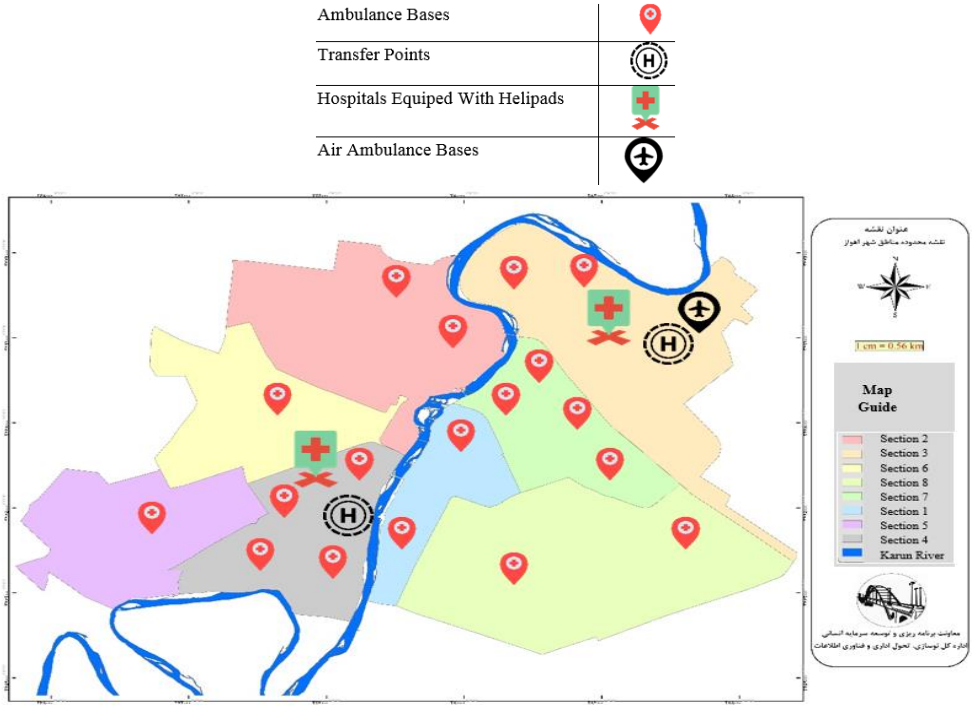


Fig 3. The locations of ambulance bases, air ambulance base, transfer points and hospitals

Table 5. The assignment of demand points to ambulance bases, transfer points and hospitals

Ambulance bases	Demand points covered
Number 1	4-7-11-16-18-23-24
Number 2	4-8-12-15-20
Number 3	6-13-14-15
Number 4	1-5-6-9-18-22-23
Number 5	9-7-2
Number 6	1-5-6-15-20
Number 7	6-9-12-15-17-19-24
Number 8	10-8
Number 9	9-11-22-23
Number 10	16-14-5
Number 11	4-7-13-16-17-22
Number 12	8-16-17-18
Number 13	1-3-9-13-17-18-19-21
Number 14	3-11-12-21
Number 15	1-3-16-20-24
Number 16	1-3-16-20-24
Number 17	1-7-17-22-24
Number 18	3-4-6-7-12
Number 19	10
Transfer point	Demand points covered
Number 1	21
Number 3	21
Hospitals	Demand points covered
Number 1	2-3-4-9-20-24
Number 2	6-7-11-12-19
Number 3	2-5-7-9-11-12-13-15-16-18-19-24
Number 4	6-10-17-24
Number 5	3-4-6-10-11-13-22
Number 6	6-14-15-16-22-23-24
Number 7	14-13-8
Number 8	1-6-13-17-20-22-23-24
Number 9	1-17-18-19-22
Number 10	1-4-7-15
Number 11	7-11
Number 12	12-14-15-17
Number 13	24-21-5

Among the proposed zones for ambulance bases, the 18th and 19th bases are located in the eastern part of Ahvaz. Considering the density of the existing bases and the population of the western part of Ahvaz, as shown in figure 3 the activation of these proposed bases seems reasonable. As can be seen in figure 3 transfer points have been activated close to Ahvaz Petroleum Hospital and Golestan Hospital, which are currently equipped with helipads, and after obtaining the results, only some of the patients in emergencies were transported to the hospital by air ambulances. After solving the model without considering the robustness and comparing the results with the robust model as it is observed in figure 4 the objective function value is 1226.930 and better than the robust state. The difference in the objective function values in the two cases is due to the attempt to reduce the deviations and guarantee for all possible scenarios in

the robust state. This difference is shown in the diagram of in Figure 4 considering the budget of 15,000,000,000 Rial.

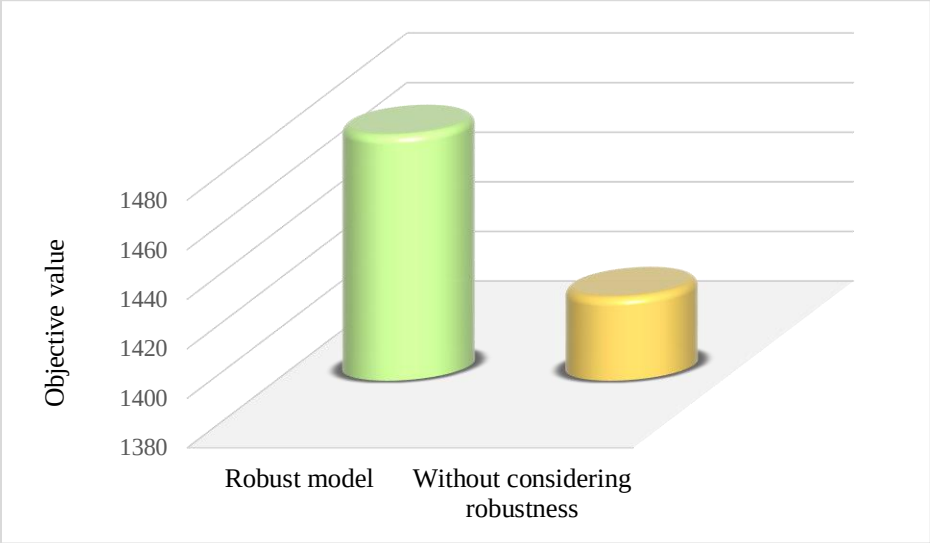
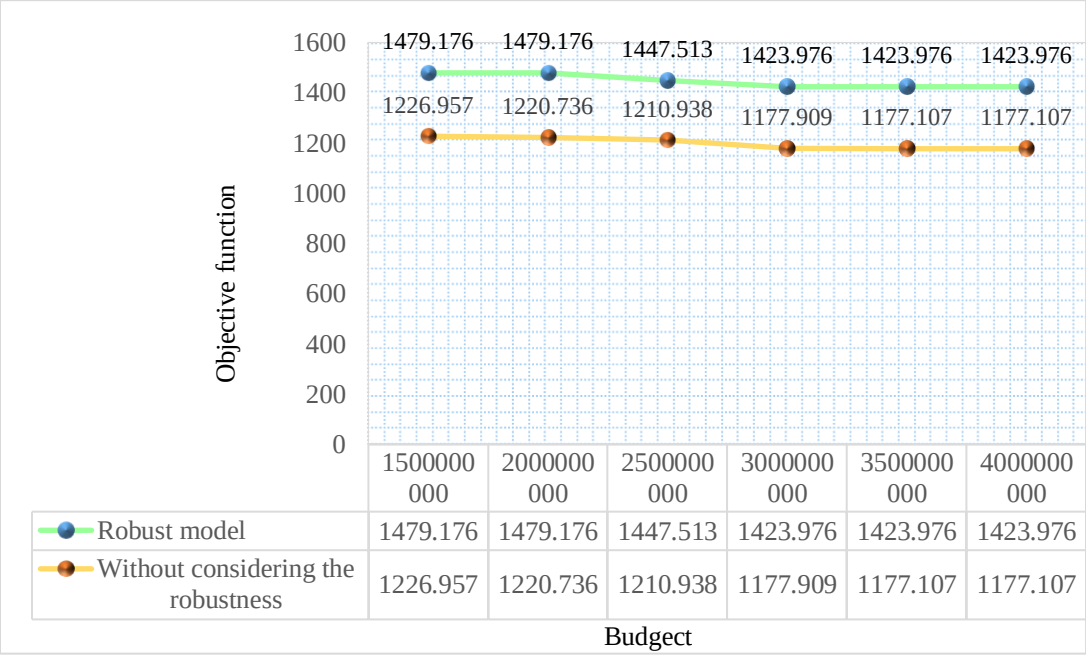


Fig 4. The difference between the value of the objective function in the robustness state and without considering the robustness

As shown in figure 5 as the budget increases, the value of the objective function decreases. However, the value of the objective function remains constant after a certain budget. That is, at a higher cost, some improvement is made in the value of the function, but after that threshold, improvement is no longer made in the objective function. Therefore, increasing the budget to 30,000,000,000, Rial is enough and it is not economical to apply more than this amount.



	1500000000	2000000000	2500000000	3000000000	3500000000	4000000000
Robust model	1479.176	1479.176	1447.513	1423.976	1423.976	1423.976
Without considering the robustness	1226.957	1220.736	1210.938	1177.909	1177.107	1177.107

Fig 5. Changes the value of the objective function by changing the budget values

As shown in figure 6 as the number of available ambulances increases, the value of the objective function decreases. Nevertheless, the value of the objective function remains constant after a regular number of available ambulances. That is, at a higher number of available ambulances, some improvement is made in the value of the function, but after that threshold, progress is no longer made in the objective function. Therefore, increasing the number of ambulances to 20 is enough and it is not economical to apply more than this number. Improvements in function could have been achieved by increasing the number of ambulances. However, achieving this optimal number of ambulances seems impractical due to problems such as high ambulance prices and budget constraints.

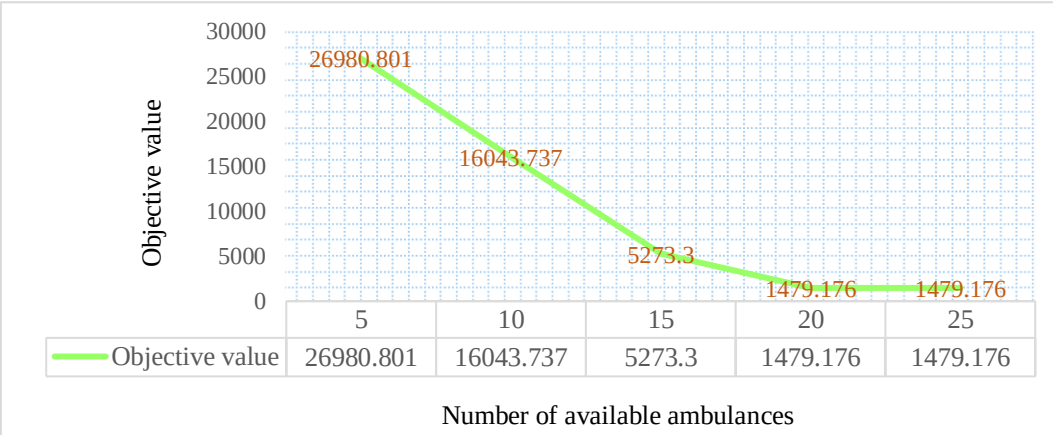


Fig 6. Changes the value of the objective function by changing the number of available ambulances

In addition, by comparing the ambulance bases built in two states, we realize that ambulance bases were activated in the eastern and densely populated areas of Ahvaz city to ensure that all scenarios are met and to meet the demand of all areas. While in the state without considering robustness, ambulance bases were activated in the eastern and western parts of the city equally. Considering table 6 as the value of μ increases, although the value of the objective function increases, the deficiency resulting from unanswered demands decreases.

Table 6. The results of the change in coefficient of model robustness in objective function

	$\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\delta_{il}^{s}$	$\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\delta'^{s}_{il}$	z
$\mu = 1$	213	0	1235. 542
$\mu = 3$	64	0	1393. 995
$\mu = 6$	28	0	1458. 581
$\mu = 9$	0	0	1479. 176
$\mu = 12$	0	0	1479. 176

5- Conclusion

EMS is principle issues of health systems and involves delivery first aid services and transferring patients to appropriate health center. This paper proposes a two-phase approach based on data envelopment analyses and robust scenario-based mathematical model to design EMS network in an uncertain environment. Applying a DEA approach, the first phase determines more valid and practical points for candidate locations. In addition, by removing the inappropriate points of the candidate locations can help to reduce the size of the problem. In the second phase, the strategic and tactical decisions of the concerned EMS is determined While the speed of air ambulances are efficiently faster than ground ambulances, the proposed optimization model takes into account the location of HEMS as well as EMS

using location of transfer points to save time and for landing and taking off airplanes and equipping hospitals with helipads. The proposed model is capable of taking into account the fairness considerations between demand areas through the objective function. A striking finding is that patients can be categorizing for allocating emergency facilities and appropriate hospitals with their related illnesses to patients. The proposed model is validated via examining a case study. Our analyses shows, it is corroborated that optimizing our EMS location problem under the robust scenario-based mathematical model formulation will incurred to open ambulance bases in the eastern part of Ahvaz with considering the density of the existing bases and the population of the western part of Ahvaz. As such, intriguing finding is that the proposed model appreciably transported only some of patients in emergencies to the hospital by air ambulances through transfer points. For further research, the proposed model can be expended on few aspects such as classifying ambulance bases on patients' status and formulating a multi-objective mathematical programming model for the concerned problem. In addition, proposing meta-heuristic and benders decomposition algorithms to deal with the complexity of the model in larger number of scenarios can be interesting avenues for future research.

References

Aringhieri, R., Bruni, M. E., Khodaparasti, S., &van Essen, J. T. (2017). Emergency medical services and beyond: Addressing new challenges through a wide literature review. *Computers & Operations Research*, 78, 349–368.

Banker, R. D., Charnes, A., &Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management science*, *30*(9), 1078-1092.

Bélanger, V., Ruiz, A., &Soriano, P. (2019). Recent optimization models and trends in location, relocation, and dispatching of emergency medical vehicles. *European Journal of Operational Research*, 272(1), 1-23.

Boujemaa, R., Jebali, A., Hammami, S., Ruiz A., &Bouchriha, H. (2018). A stochastic approach for designing two-tiered emergency medical service systems. *Flexible Services and Manufacturing Journal*, 30(1-2), 123–152.

Charnes, A., Cooper, W. W., &Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research*, *2*(6), 429-444.

Dehghani, E., Jabalameli, M. S., &Jabbarzadeh, A. (2018). Robust design and optimization of solar photovoltaic supply chain in an uncertain environment. *Energy*, *142*, 139-156.

Dehghani, E., Jabalameli, M. S., Naderi, M. J., &Safari, A. (2020). An environmentally conscious photovoltaic supply chain network design under correlated uncertainty: A case study in Iran. *Journal of Cleaner Production*, 262, 1-48.

Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society: Series A (General)*, *120*(3), 253-281.

Furuta, T., &Tanaka, K. I. (2013). Minisum and minimax location models for helicopter emergency medical service systems. *Journal of the Operations Research Society of Japan*, 56(3), 221-242.

Hosseinijou, S. A., &Bashiri, M. (2012). Stochastic models for transfer point location problem. *The International Journal of Advanced Manufacturing Technology*, 58(1-4), 211-225.

Kalantari, H., Yousefli, A., Ghazanfari, M., &Shahanaghi, K. (2014). Fuzzy transfer point location problem: a possibilistic unconstrained nonlinear programming approach. *The International Journal of Advanced Manufacturing Technology*, 70(5-8), 1043-1051.

Leung, S. C., Tsang, S. O., Ng, W. L., &Wu, Y. (2007). A robust optimization model for multi-site production planning problem in an uncertain environment. *European journal of operational research*, 181(1), 224-238.

Li, X., Zhao, Z., Zhu, X., &Wyatt, T. (2011). Covering models and optimization techniques for emergency response facility location and planning: a review. *Mathematical Methods of Operations Research*, 74(3), 281-310.

Liu, K., Li, Q., &Zhang, Z. H. (2019). Distributionally robust optimization of an emergency medical service station location and sizing problem with joint chance constraints. *Transportation research part B: methodological*, 119, 79-101.

Mohamadi, A., &Yaghoubi, S. (2017). A bi-objective stochastic model for emergency medical services network design with backup services for disasters under disruptions: An earthquake case study. *International journal of disaster risk reduction*, 23, 204-217.

Mohammadi, R., Ghomi, S. F., &Jolai, F. (2016). Prepositioning emergency earthquake response supplies: A new multi-objective particle swarm optimization algorithm. *Applied Mathematical Modelling*, 40(9-10), 5183-5199.

Mulvey, J. M., Vanderbei, R. J., &Zenios, S. A. (1995). Robust optimization of large-scale systems. *Operations research*, 43(2), 264-281.

Nickel, S., Reuter-Oppermann, M., &Saldanha-da-Gama, F. (2016). Ambulance location under stochastic demand: A sampling approach. *Operations Research for Health Care*, 8, 24-32.

Sabouhi, F., Tavakoli, Z. S., Bozorgi-Amiri, A. &Sheu, J. B. (2019). A robust possibilistic programming multi-objective model for locating transfer points and shelters in disaster relief. *Transportmetrica A: transport science*, 15(2), 326-353.

Shahriari, M., Bozorgi-Amiri, A., Tavakoli, S., &Yousefi-Babadi, A. (2017). Bi-objective approach for placing ground and air ambulance base and helipad locations in order to optimize EMS response. *The American journal of emergency medicine*, 35(12), 1873-1881.

Shen, W. F., Zhang, D. Q., Liu, W. B., &Yang, G. L. (2016). Increasing discrimination of DEA evaluation by utilizing distances to anti-efficient frontiers. *Computers & Operations Research*, *75*, 163-173.

Tofighi, S., Torabi, S. A. &Mansouri, S. A. (2016). Humanitarian logistics network design under mixed uncertainty. *European Journal of Operational Research*, 250(1), 239-250.

Yu, C.S., &Li, H. L. (2000). A robust optimization model for stochastic logistic problems. *International journal of production economics*, 64(1-3), 385-397.

Zarrinpoor, N., Fallahnezhad M. S., &Pishvaee, M. S. (2018). The design of a reliable and robust hierarchical health service network using an accelerated Benders decomposition algorithm. *European Journal of Operational Research*, 265(3), 1013-1032.

Zhang, Z. H., &Jiang, H. (2014). A robust counterpart approach to the bi-objective emergency medical service design problem. *Applied Mathematical Modelling*, 38(3), 1033-40.