Journal of Industrial and Systems Engineering
Vol. 13, No. 1, pp. 111-135
Winter (January) 2020

A reward-penalty stochastic pricing and advertising model under demand uncertainty

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Abstract

This paper aims in assessing the effects of governmental policies on a maximal covering location problem facing stochastic demand which is sensitive to both the retail price and facility advertising efforts. A reward-penalty two-stage stochastic programming model is proposed to formulate the problem as a supportive approach in a mixed integer non-linear programming form. In particular, a stochastic pricing and advertising dependent demand model in a facility location configuration is developed which sets the retail price for each opened facility and various advertising effort levels based on the zone's attractiveness. To promote customer welfare and satisfaction, the legislative counterpart of the reward-penalty model is introduced. The legislative model assigns the minimum satisfaction demand level to the model as a constraint. In both models, the firm tries to maximize its net profit according to government decisions. An analytical method based on the L-shaped algorithm is provided to determine the best solutions of the first and second stages with coping nonlinearity term of the proposed models. Finally, numerical examples are developed to illustrate the governmental policies impacts to reach to the most social welfare as well as the least reward-penalty legislation.

Keywords: Maximal covering location problem, pricing, advertising efforts, stochastic demand, government, L-shaped algorithm.

1-Introduction

Pricing and advertising efforts are key tools that enable firms to enhance their profit by focusing on knowing the consumer behavior (Asamoah and Chovancová 2011, Bashir and Malik 2010). Large firms invest amounts of money in recognizing the characteristics of their own and potential customers. In economic theory, the most useful instrument for this purpose is the demand function. The demand function shows the quantity demanded by customers in a given market as a function of price, income level, advertising, social conformity and nonconformity and others (Shy 2008).

In this study, the demand function is considered to be influenced by the retail price and advertising effort levels in a maximal covering location problem (MCLP). MCLP is a particular kind of facility location problem (FLP) that a predefined number of facilities must be located among potential locations by a single decision-maker to maximize the covered demand points (Fischetti, et al. 2016; Schlicher, et al. 2017).

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ISSN: 1735-8272,

Covering location model has been proven to be useful in many research areas e.g. location emergency facilities, communication networks, retail stores (Aziz, et al. 2019, Li, et al. 2011; Lee and Murray 2010; Plastria and Vanhaverbeke 2007).

Although, in most of the papers the MCLP is investigated from the companies view to capture more market share, it may become one of the social concerns and important challenges of the governments when a necessary product supply is not enough. Indeed, the unwillingness of companies to engage in production activities due to the governmental price cap strategies, low prices, high costs and also high uncertainties in quantity demands may lead governments to use motivational and compulsory tools. In the other words, in some cases or some situations, it may not be profitable for companies to engage in production activities or they prefer to decrease their production because of the low price and production related costs, such as opening facilities, transportation costs, and advertising effort payments.

Thus, in this paper, a maximal covering location problem is developed to determine retail stores among demand points according to the government incentive policies about production regulation. Since the facility location designer policies have vital effects on social welfare and sustainability; governments enact their supportive policies through incentive/punitive mechanisms to lead firms as a superior organization to ensure satisfaction of the desired proportion of customer demand. Of course, governments should carefully consider all the implications of enacting rules and regulations to ensure that they are appropriate for sub-organizations and provide social welfare enough. In this paper, social welfare is represented by the amount of responded demand.

For this purpose, a reward-penalty mechanism is applied in assessing the governmental policies impact on pricing and advertising decisions of a maximal covering location problem as well as the social welfare in an uncertain environment in two configurations based on the government role; 1- supportive role and 2- legislative role.

Thus, the proposed model is more suitable for products with low price and low cost savings; while companies are not interested in producing products and governments should intervene due to the social issues, as a superior and legislative entity to force or motivate companies to produce the desired level of the products.

Figure 1 shows a general structure of the proposed problem. In this figure, bold red nodes represent the opened retail stores and bold blue nodes show the demand nodes and circles are the coverage demand zone by each facility.

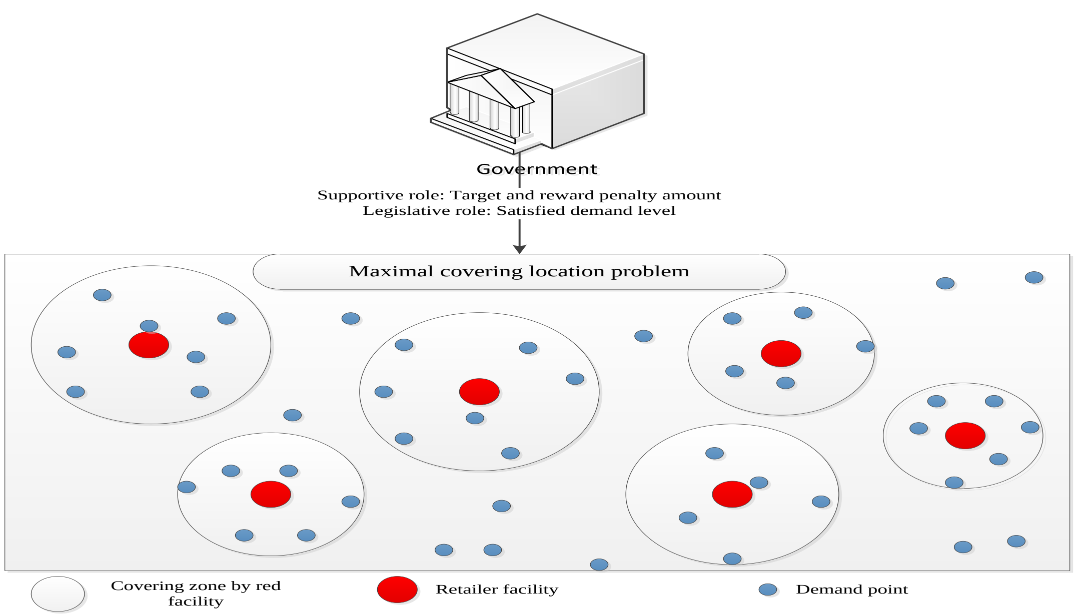

Fig 1. A schematic view of the stochastic facility location model

As shown in figure 1, some fixed demand nodes exist in the market and each company should determines the locations of the retailer facilities based on the each facility covering zone, company profit and government policies.
In addition, customer demand functions are influenced by pricing and advertising effort levels as well as a random component. To cope with this uncertainty, a two-stage stochastic programming model in an MCLP configuration is proposed.

The rest of the paper is organized as follows: The literature review is presented in section 2. In section 3, the reward-penalty facility location problem model definition and formulation are developed. The L-shaped algorithm is proposed to solve the proposed two-stage stochastic programming model in section 4. In the next section, computational results are analyzed. Finally, conclusions and suggestions for future research are presented in the last section.

2-Literature review

The MCLP is a classic optimization model from the location science literature (Church and Velle 1974). Various strategic and operational aspects of MCLP have been investigated in the last decades. In this study, it is attempted to propose a new formulation to deal with pricing and advertising decisions in an MCLP model by considering government decision impact as a superior authority in a stochastic environment. Thus, the focus of the literature survey in this study is on the MCLP model with demand uncertainty, advertising and pricing dependent demand models and governmental regulation interferences in an MCLP model.

2-1- Facility location problem under uncertainty

Ignoring of the inherent uncertainty in the facility location problem parameters can lead to inferior quality and less realistic results. Thus, in most of the recent and relevant studies, FLP and MCLP are designed in an uncertain environment. Uncertainty in the demand parameter is most common. Plastria and Vanhaverbeke (2009), Albareda-Sambola, et al. (2011), Berman and Wang (2011), Alizadeh (2013), Alizadeh, et al. (2015), Bieniek (2015), Vatsa and Jayaswal (2016), Zhang, et al. (2017a), and Correia, et al. (2018) have studied uncertainty in demand in facility location problem based on the scenario.

Besides, Albareda-Sambola, et al. (2011) studied a capacitated stochastic facility location problem with Bernoulli demands in a two-stage stochastic programming configuration. Alizadeh, (2013) and Alizadeh, et al. (2015) presented a capacitated location-allocation problem under stochastic customer demands based on Bernoulli distribution function. Bieniek (2015) proposed a single capacitated facility location model with stochastically distributed demands under various demand distribution functions. Besides, Nickel, et al. (2012), Albareda-Sambola, et al. (2013), Hosseini, and MirHassani (2015), Habibzadeh, et al. (2016), considered a multi-period facility location problem under demand uncertainty. We refer the readers for a detailed review on facility location problem under uncertainty to Snyder, (2006); Correia, (2015); Gülpınar, (2013), Farahani, et al. (2012), and Ortiz-Astorquiza, et al. (2017).

It can be concluded that facility designers have to deal with uncertainty in the demand parameter while markets have become more competitive, transparent and agile. Thus in this paper, a stochastic scenario-based MCLP model is developed by considering uncertainty in the demand parameter.

2-2- Pricing and advertising dependent demand models

Although advertising activities gained more than $300 billion in the United States in 2010 and more than $500 billion in the world, few studies have addressed the demand function in facility location models (Statistics & Facts on the U.S. Advertising Industry). Based on the report of the IHS Markit company, which was published in March 2015, the US economy spent $36.7 trillion on sales activity in 2014. And, IHS estimates $2.4 trillion in direct sales were stimulated as a result of the $297 billion that companies spent on advertising for their products and services. Thus, approximately 6.5% of US sales activity is directly stimulated by advertising (IHS ECONOMICS AND COUNTRY RISK, 2015).

Besides, most of the recent related studies have considered pricing and advertising decisions in a supply chain based on game theory approaches (Gutierrez, et al. 2019, Xiao, et al. 2019). All of the studies in this area emphasized that higher advertising levels can increase demand. We refer readers for a comprehensive review of advertising efforts to Araman and Popescu (2010), and Wu, et al. (2011).

The current study aims to determine optimal decisions on the pricing and advertising level simultaneously in a facility location model. However, only a few scholars have studied the supply chain design problem incorporating joint decisions on pricing and advertising. Ray (2005) addressed a model with random demand which is sensitive to both price and non-price factors. Helmes and Schlosser (2013), and Schlosser (2016, 2017) addressed stochastic dynamic pricing and advertising model with constant demand elasticity based on the game theory approach. Rad, et al. (2016), Maiti and Giri (2017), and Liu, et al. (2014) studied a two-stage supply chain design with price and advertisement dependent demand based on game theory approach. Liu et al. (2014), investigated an inventory decision under price and advertising dependent demand by considering customer welfare. Gou, et al. (2020) investigated the role of local media companies in co-op advertising programs and differential pricing strategy on a system consisting of a manufacturer, a retailer, and a local media company.

2-3- Government regulation in facility location problem model

In recent decades, the government as a legislative and authorized entity tries to lead organizations to provide social welfare and sustainability as more as possible. Kulshreshtha and Sarangi (2001), Wojanowski, et al. (2007) developed a model to analyze the impact of governmental incentive policy based on deposit-refund systems for collecting and recycling. Sheu, et al. (2005), Mitra and Webster (2008), Aksen, et al. (2009) proposed a model by considering governmental subsidies for product recovery. Plambeck and Wang (2009), found that applying the “fee upon disposal” policy motivates manufacturers to design for recyclability. Sheu and Chen (2012), analyzed the effect of green taxation and subsidization as governmental financial interventions on green supply chain profits and social welfare. Shutao and Jiangao (2011), analyzed the effect of government’s checking, reward-penalty mechanism, transportation costs and some policy-making suggestions on energy saving behavior of the industries. Wang, et al. (2017), Wang, et al. (2015), and Amini, et al. (2014) proposed a reward-penalty mechanism as one on of the government interposition is considered to motivate recycling. Yang and Xiao (2017) developed a three-game theory-based models to cope with green supply chain problem with governmental interventions under uncertain parameters. Zhang, et al. (2017b), explored the impact of government intervention on waste cooking oil to an energy company. Zhao, et al. (2017), explored the impact on market incentive and government regulation on vegetable farmer’s behavior. Brawley (2017), addressed the role of government regulation on cancer prevention. Niu, et al. (2017), explored the impact of government regulation on fashion procurement strategies in punishing and subsidizing forms.

A summary of related studies on facility location problem (FLP) is compared in table 1 in terms of model configuration, government policy, considering pricing and advertising polices, and uncertainty approach.

Table 1. A summary of the recent studies on facility location problem

Reference	Model		Governmental policy consideration		features		Uncertainty	
	FLP	**MCLP**	**supportive**	**legislative**	**pricing**	**advertising**	**parameter**	**approach**
Gulpnar, et al. (2013)	✓		-	-	-	-	demand	Robust optimization
Hosseini & Mirhassani (2015)		✓	-	-	-	-	serve traffic flow	two-stage stochastic
Bieniek (2015)	✓		-	-	-	-	demand	Stochastic distribution
Mestre, et al. (2015)	✓		-	-	-	-	demand	Scenario-stochastic based
Vatsa & Jayaswal (2016)		✓	-	-	-	-	Server (doctors)	Scenario-stochastic based
Zhang, et al. (2017a)		✓	-	-	-	-	demand	Chance constraint
Zhang, et al. (2017b)	✓			fee and penalty	-	-	-	-
Mišković, et al. (2017)	✓		-	-	-	-	transportation costs	stochastic & Robust optimization
Correia, et al. (2018)	✓		-	-	-	-	demand	Scenario-stochastic based
This study		✓	✓	✓	✓	✓	demand	Scenario-stochastic based

As shown in table 1, despite the fact that the concept of "customer welfare" and "service level" has attracted the attention of the researchers, in all variants of MCLP we couldn't find any direct effort to study the impact of the government intervention on the covering location problem configuration. Based on the above discussion, pricing and advertising effort consideration, supportive and legislative government intervention and demand function uncertainty constitute collectively a significant departure from the current studies in the covering location problem.

Thus, this paper studies the strategic behavior of a firm under supportive and legislative government intervention under demand uncertainty. Two reward-penalty two-stage stochastic programming models are provided to evaluate the effects of governmental policies in a supportive and legislative role. In both models, the firm tries to maximize its net profit according to government decisions. An analytical method based on the L-shaped algorithm is provided to cope with the nonlinearity term of the models. The firm's profit maximization and satisfying social welfare target imposed by the government are examined through some numerical examples.

3-Model definition and formulation

3-1-Problem definition

In this paper, a MCLP is proposed in a two-stage stochastic programming using a reward-penalty mechanism. As shown in figure 1, a facility location model under demand uncertainty is designed with aiming to achieve the most profit under governmental legislative and supportive policies. According to this model, predefined N facilities of retailers must be located among customer's zones J to respond to customer's demands. The opened facility locations face a random demand that is sensitive to both the retail price and facility advertising efforts. By knowing the characteristics of demands, retailers need to

set prices for each opened facility and various advertising efforts for each customer's zone based on its attractiveness and costs for the retailers which are reflected as sales effort's cost in the proposed model. Besides, the facility location decisions are made according to the governmental regulations as a reward-penalty mechanism. The government tends to achieve the most social welfare by leading the facility retailers to respond as high as the customer demands by the least reward-penalty amount.
The sequence of the events in the proposed two-stage stochastic model is as follows:

(1) The government offers the retailers a reward/penalty contract for a new year in a supportive role and fixes a satisfied demand level for retailers in a legislative role.

(2) Retailers choose facility locations (X_i) among the customer's zones and determine desirable order quantity (Q_i), advertising effort levels (e_{ij}) and prices (p_i), according to the government policies.

(3) The selling season starts and the stochastic component of the demand function (ξ) is observed.

(4) The products transfer between retailers and demand zones (Y_{ij}^{ξ}) and also the payments based on the agreed contract transfer between government and retailers.

A schematic view of the two-stage stochastic programming model is presented in figure 2.

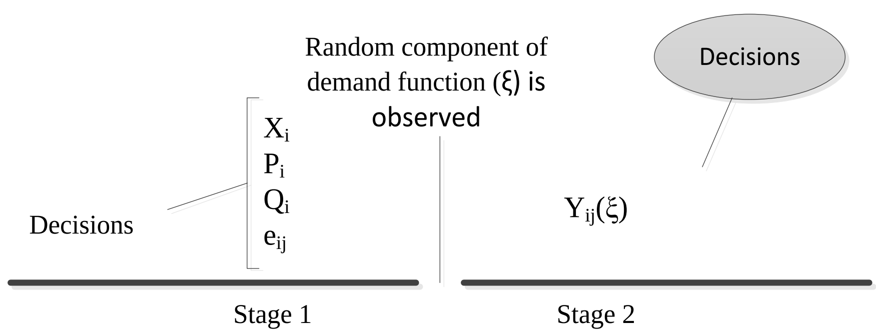

Fig 2. A schematic view of two-stage stochastic stages

The main assumptions of the studied problem are listed as follows:

- A single product maximal covering location problem is proposed.
- The firm knows the decisions of the government before making any own decision, thus, government decisions are considered as parameters in the proposed model.
- A finite number of candidate locations are considered for the establishment of retailer facilities.
- The demand function is affected by retailer price, advertising effort, and a random component as a linear function.
- Transportation cost per unit of transport product is proportional to the Euclidean distance.

3-2- Problem formulation

In this section, a two-stage stochastic programming facility location with price and advertising level effort dependent demand model formulation is presented. The following notations are used in the model formulation.

Indices:

I : Set of potential retailer's facility locations, ($i = 1, 2, ..., I$)

J : Set of fixed locations of demand zones, ($j = 1, 2, ..., J$)

Parameters:

N: The number of the retailer facilities.
C: Purchasing cost of new product for the retailers.
T: Sales target of the government.
λ: The reward or penalty for unit of product.
V: Salvage value of remained products.
$\widetilde{D}_{ij}$: Random variable of customer demand at zone j for facility retailer i.
dis_{ij}: Euclidean distance between retailer facility in potential location i and demand zone j.
K: Transportation unit cost
Cap_i: Capacity of retailer facility i
D_j^{max}: Maximum potential demand in demand zone j
p_i^{max}: Maximum price that retailer facility i can pay
ξ: Random demand component
P_ξ: Probability of ξ accrued

Decision variables:

X_i: Binary variable which is equal to 1 if a retailer facility is opened at zone i ; 0, otherwise.
p_i: Retail price of product for retailer facility i.
e_{ij}: Advertising effort level of retailer's facility i for demand zone j.
Q_i: Order quantity in facility location i.
Y_{ij}^{ξ}: Flow of products between retailer facility i and demand zone j under demand uncertainty ξ .
θ: Auxiliary variable for Master problem which represents second stage part of the objective function
OCQ_i^v: Coefficient value of variable Q_i in optimality cut v
OCp_i^v: Coefficient value of variable p_i in optimality cut v
OCe_{ij}^v: Coefficient value of variable e_{ij} in optimality cut v
ORH^v: Right hand side of optimality cut v
FCQ_i^r: Coefficient value of variable Q_i in feasibility cut r
FCp_i^r: Coefficient value of variable p_i in feasibility cut r
FCe_{ij}^r: Coefficient value of variable e_{ij} in feasibility cut r
FRH^r: Right hand side of feasibility cut r
ω_{ij}^v: Dual value of constraint (21) in constructing optimality cut v
τ_i^v: Dual value of constraint (22) in constructing optimality cut v
δ_j^v: Dual value of constraint (23) in constructing optimality cut v
π_{ij}^r: Dual value of constraint (25) in feasibility check problem in constructing feasibility cut r
σ_j^r: Dual value of constraint (26) in feasibility check problem in constructing feasibility cut r
ρ_i^r: Dual value of constraint (27) in feasibility check problem in constructing feasibility cut r

The proposed two-stage stochastic location facility model is aiming to achieve the highest retailer's profit which is obtained by subtracting income values and costs. Incomes are based on sales and salvage values as follow:

$$Income = \sum_{\xi} P_\xi \left(\sum_i \sum_j p_i Y_{ij}^{\xi} + \sum_i \sum_j V \left(Q_i - Y_{ij}^{\xi} \right) \right) \tag{1}$$

And costs are determined according to the purchases, transportations and advertising effort level's costs as:

$$Cost = \sum_i CQ_i + \sum_{\xi} P_{\xi}\left(\sum_i \sum_j Kdis_{ij}Y_{ij}^{\xi}\right) + \sum_i \sum_j g(e_{ij}) \tag{2}$$

In the above formulation, g(e_{ij}) is assumed to be the retailer's costs of exerting an effort level e_{ij} and is considered g(e_{ij}) $= \mu e_{ij} dis_{ij}^2$ in the proposed model where is set as an advertising constant coefficient. According to this formulation, advertising effort costs of the far zones are more than the near one. Thus, nears are more attractive to be serviced.

Besides, the reward-penalty amount can be considered as an income if customer demand responds more than government target and otherwise sets as a cost component. Thus, the model objective function is displayed in equation (3):

$$\pi = \sum_{\xi} P_{\xi}\left(\sum_i \sum_j p_i Y_{ij}^{\xi} + \sum_i \sum_j V\left(Q_i - Y_{ij}^{\xi}\right)\right) - \sum_i CQ_i + \sum_{\xi} P_{\xi}\left(\sum_i \sum_j \lambda\left(Y_{ij}^{\xi} - T\right)\right) - \sum_{\xi} P_{\xi}\left(\sum_i \sum_j Kdis_{ij}Y_{ij}^{\xi}\right) - \sum_i \sum_j g(e_{ij}) \tag{3}$$

It should be mentioned that customer's demand is denoted by $\widetilde{D}_{ij}$ with probability density function of given price and effort levels, $f\left(D_j \middle| (p_i, e_{ij})\right)$. Also, we assume that demand is stochastically increasing in efforts and decreasing in price, i.e., $\frac{\partial F\left(D_j \middle| (p_i, e_{ij})\right)}{\partial e} > 0$ and $\frac{\partial F\left(D_j \middle| (p_i, e_{ij})\right)}{\partial p} < 0$ (He, et al. 2009). We further specify the demand function as $\widetilde{D}_{ij} = L_{ij}\left(p_i, e_{ij}\right) + \xi$. ξ is assumed as the random component of the demand with density function $\varphi(\xi)$. Let we arbitrarily choose a linear demand function such as $L_{ij}\left(p_i, e_{ij}\right) = (\alpha - \beta p_i + \gamma e_{ij})$ for the $L_{ij}\left(p_i, e_{ij}\right)$ component of the demand function term and a uniformed distribution for random component, $\sim Uniform(A, B)$ (C. Petruzzi and Dada 1999).

Moreover, in order to obtain price and advertising sales efforts inelastic regions, price and effort levels elasticity should be greater than 1 (Shy 2008). Thus, according to the elasticity formulation follows constraint should be considered in the proposed model:

$$|elasticity(p)| = \left|\frac{\partial D_{ij}}{\partial p} * \frac{p_i}{D_{ij}}\right| \geq 1 \rightarrow D_{ij} \geq \frac{\alpha + \gamma e_{ij}}{2}$$

$$\left|elasticity(e_{ij})\right| = \left|\frac{\partial D_{ij}}{\partial e_{ij}} * \frac{e_{ij}}{D_{ij}}\right| \geq 1 \rightarrow \beta p_i - \alpha \geq 0.$$

In terms of the above-mentioned notations, the proposed mixed-integer non-linear programming (MINLP) model can be formulated as follows in a supportive role of the government:

$$Max\ \pi = \sum_i \sum_j \sum_{\xi} P_{\xi}(p_i - V + \lambda - Kdis_{ij})Y_{ij}^{\xi} + \sum_i (V - C)Q_i - \lambda T - \sum_i \sum_j \mu e_{ij} dis_{ij}^2 \tag{4}$$

$$Y_{ij}^{\xi} \leq (\alpha - \beta p_i + \gamma e_{ij}) + \xi \qquad \forall i, j, \xi \tag{5}$$

$$\sum_i X_i = N \tag{6}$$

$$Q_i \leq X_i Cap_i \qquad \forall i \tag{7}$$

$$Q_i \geq \sum_j Y_{ij}^{\xi} \qquad \forall i, \xi \qquad (8)$$

$$\sum_i Y_{ij}^{\xi} \leq D_j^{max} \qquad \forall j, \xi \qquad (9)$$

$$2(\alpha - \beta p_i + \gamma e_{ij}) \geq \alpha + \gamma e_{ij} \qquad \forall i, j \qquad (10)$$

$$p_i \geq \frac{\alpha}{\beta} * X_i \qquad \forall i \qquad (11)$$

$$p_i \leq p_{max} * X_i \qquad \forall i \qquad (12)$$

$$Q_i \geq 0, e_{ij} \geq 0, Y_{ij}^{\xi} \geq 0, X_i \in \{0,1\} \qquad \forall i, j, \xi \qquad (13)$$

Equation (4) represents the facility location objective function which is to maximize the facility location-allocation's model profit. Constraint (5) represents each customer's demand function. Equation (6) ensures the model to open a predefined retailer facility number. Constraints (7) and (8) express the capacity restrictions of each opened retailer facilities to entries and exits. Constraint (9) ensures each demand zone amount served restriction. Constraints (10) and (11) ensure the price and advertising sales efforts choose inelastic regions. Finally, each retailer's price upper bound and variable types are declared in constraint (12) and (13), respectively.

On the other hand, if the government considered as a legislative entity in the proposed model, the minimum satisfaction demand level will be assigned to the model as a constraint. Thus, the proposed mixed-integer non-linear programming (MINLP) model can be rewritten as follow:

$$Max\ \pi = \sum_i \sum_j P_{\xi}(p_i - V - Kdis_{ij})Y_{ij}^{\xi} + \sum_i (V - C)Q_i - \sum_i \sum_j \mu e_{ij} dis_{ij}^2 \qquad (14)$$

$$\sum_i Y_{ij}^{\xi} \geq L \sum_i (\alpha - \beta p_i + \gamma e_{ij} + \xi) \qquad \forall i, j \qquad (15)$$

(5)-(13)

In this model, the government set a predefined satisfied demand level (L) for retailers as a constraint against a reward penalty model, which is represented in constraint (15).

Where L presents the desirable ratio of responded demand by the government and it is as equation (16):

$$L = \frac{T}{\sum_j D_j^{max}} \qquad (16)$$

It is worth to be mentioned, this regulation may affect the facility location feasibility solution space. Thus, the probability of model infeasibility will be increased against the reward-penalty model.

4-Solution methodology

Since the proposed model is a mixed-integer nonlinear programming (MINLP) model, commercial solvers such as BARON on GAMS is not efficient to solve the model in large size problem. The nonlinearity term of the proposed model is $\sum_i \sum_j p. Y_{ij}^{\xi}$ in the objective function which is a production of two continues variables. Thus, commercial solvers like BARON which are appropriated for non-linear models are suitable for small size problems to obtain global solutions. In this section, the L-shaped algorithm is applied to deal with non-linearity term by dividing the MINLP model to a mixed-integer linear programming (MIP) and a linear programming (LP) models due to solving first and second stages of the proposed model separately. Indeed, the L-shaped algorithm copes with non-linearity by dividing the two-stage stochastic programming model in two decomposed models, a master problem with MIP feature and a sub-problem which is LP.

The master problem represents the first stage of the model which is influenced by an approximation of the second stage objective function. And, the sub-problem is the second stage of the proposed stochastic model which is used to improve the approximation term of the master problem in some steps. Since the nonlinearity term of the proposed model is a production of the first stage variable (p) and a second stage one (Y_{ij}^{ξ}), the L-shaped algorithm is efficient to solve the proposed model by decomposing it to a MIP and a LP models.

4-1- L-shaped algorithm

In this section, the L-shaped algorithm is considered to solve the proposed two-stage stochastic programming model. Slyke and Wets have studied L-shaped linear programming and have proposed an algorithm for it. They applied the algorithm to linear optimal control and two stage stochastic linear programming (Slyke and Wets 1969). This algorithm is a decomposition method that is useful for solving stochastic problems which is used in many previous papers (Bentaha, et al. 2013, Lei, et al. 2014, Biesinger, et al. 2016, Li and Grossmann 2018, Placido dos Santos and Oliveira 2019, Mirzapour Al-e-hashem, et al. 2019).

The main idea of the L-shaped algorithm is to approximate the non-linear term in the objective. Since the nonlinear objective term involves a solution of the second-stage solution, decomposing the two-stage stochastic programming model can be useful to cope with non-linearity. Thus, the L-shaped method is applied to solve the proposed model in which the MIP in the preparedness stage is considered as the master problem and the LP in the response stage is decomposed into several sub-problems based on discrete demand scenarios. The optimal solution could be found through the iterations between the master problem and the sub-problem in each scenario.

The brief steps of the L-shaped algorithm are presented as follow for the proposed model. Consider r is the number of feasibility cut and v is the number of optimality cut and t is the sign of iterations.

Step 0: Set r=0, t=1, v=0, S=0, and θ=+∞;

Step 1: Solve master problem and determine first stage variables ($X_i(t)$,$Q_i(t)$, $p_i(t)$, $e_{ij}(t)$);

Step 2: If s=S go to step 5 otherwise set s=s+1 and solve FCP under scenario s by considering the first stage variables as parameters;

Step 3: If the objective function of FCP equals to zero go to Step 2 otherwise go to step 4.

Step 4: Set r=r+1, s=0 and calculate parameters for feasibility cut r and return to step 1.

Step 5: Set s=0 and set first stage variables as parameters in primal sub-problem until s=S **Step 6:** If the second stage objective > θ, obtain the optimality cut v and return to step 1 otherwise the optimal solution is obtained.

4-1-1- The master problem

According to the main model (constraints (4)-(13)), the first stage variables that impact the second stage part of the model take coefficient in the construction of optimality and feasibility cuts. Consider the following rewritten proposed model as a Master problem. The objective of the master problem includes the total benefit in the preparedness stage and the upper bound θ of the objective function of the response stage.

$$Max\, \pi = \sum_i (V-C)Q_i - \lambda T - \sum_i \sum_j \mu e_{ij} dis_{ij}^2 + \theta \tag{16}$$

$$\theta \leq \mathrm{ORH}^v + \mathrm{OCp}_i^v + \mathrm{OCQ}_i^v + \mathrm{OCe}_{ij}^v \qquad \forall v \tag{17}$$

$$\mathrm{FCe}_{ij}^{r} + \mathrm{FCp}_{i}^{r} + \mathrm{FCQ}_{i}^{r} + \mathrm{FRH}^{r} \leq 0 \qquad \forall r \qquad (18)$$

Constraints (6), (7), (10), (11) and (12)

$$Q_i \geq 0, e_{ij} \geq 0\ , X_i \in \{0,1\} \qquad \forall i,j \qquad (19)$$

Constraints (17) and (18) represent the vth (=1,..., v^*) optimality cut and the rth (=1,..., $r^*(t^*)$) feasibility cut, respectively which are presented in section 4.1.3.

4-1-2- The primal sub-problem

To obtain improvement in the approximation of the θ in some steps, primal sub-problem should be solved for each demand scenario separately by considering the solution (X_i, Q_i, p_i, e_{ij}) obtained from the master problem as a parameter. Variable θ in the master problem is seen as the upper bound of the primal sub-problem, and its initial value is $+\infty$. In the first iteration (t=1), due to the fact that there are no feasibility cuts and optimality cuts, we only solve the part of the master problem (except constraints (17) and (18)), and substitute the solution into the primal sub-problem. From the second iteration onward, in each iteration, a new cut will be added to the master problem, and the value of r^*or t^* will increase correspondingly. Primal sub-problem can be rewritten as follows:

$$Max\ \pi = \sum_i \sum_j P_{\xi}(\widehat{p_i} - V + \lambda - Kdis_{ij})Y_{ij}^{\xi} \qquad (20)$$

$$Y_{ij}^{\xi} \leq (\alpha - \beta\widehat{p_i} + \gamma\widehat{e_{ij}}) + \xi \qquad \forall i,j \qquad (21)$$

$$\widehat{Q_i} \geq \sum_j Y_{ij}^{\xi} \qquad \forall i \qquad (22)$$

$$\sum_i Y_{ij}^{\xi} \leq D_j^{max} \qquad \forall j \qquad (23)$$

$$Y_{ij}^{\xi} \geq 0 \qquad \forall i,j \qquad (24)$$

The dual of the sub-problem is used to obtain feasibility cuts and optimality cuts. To be more specific, when the primal sub-problem is feasible, the v^{th} optimality cut (Constraints (17)) is obtained. If the primal sub-problem is infeasible, the dual sub-problem is unbounded. Therefore, the set of extreme dual rays is introduced to obtain the feasibility cut.

The formulations optimality and feasibility sub-problems of the L-shaped algorithm are presented as follows:

4-1-3- Feasibility and optimality cuts

In the formulation of the feasibility check problem, auxiliary variables would be needed in constraints that contain first stage variables. In fact, when the dual sub-problem is unbounded, we obtain extreme dual rays and add feasibility cut, represented as inequalities (18) into the master problem. The FCP is rewritten in the following model (equation (25) to constraint (29)).

Where $k_{ij}^{+}, k_{ij}^{-}, L_j^{+}, L_j^{-}, V_i^{+}, V_i^{-}$ are auxiliary variables.

$$Min\ Z = \textstyle\sum_i \sum_j (k_{ij}^{+} + k_{ij}^{-}) + \sum_j (L_j^{+} + L_j^{-}) + \sum_i (V_i^{+} + V_i^{-}) \qquad (25)$$

$$Y_{ij}^{\xi} + k_{ij}^{+} - k_{ij}^{-} \leq (\alpha - \beta p_i + \gamma e) + \xi \qquad \forall i,j,\xi \qquad (26)$$

$$\sum_{i} Y_{ij}^{\xi} + L_j^{+} - L_j^{-} \leq D_j^{max} \qquad \forall j, \xi \tag{27}$$

$$Q_i \geq \sum_{j} Y_{ij}^{\xi} + V_i^{+} - V_i^{-} \qquad \forall i, \xi \tag{28}$$

$$Y_{ij}^{\xi}, V_i^{+}, V_i^{-}, L_i^{+}, L_i^{-}, k_{ij}^{+}, k_{ij}^{-} \geq 0 \qquad \forall i, j, \xi \tag{29}$$

If $\sum_i \sum_j \pi_{ij}^s(\alpha - \beta p_i + \gamma e + \xi) + \sum_j \sigma_j^s(D_j^{max}) + \sum_i \rho_i^s(Q_i) > 0$, Then feasibility cut (18) should be added to the Mater problem for each scenario *s* where:

$$\mathrm{FRH}^r = \sum_{j} \sigma_j^s(D_j^{max}) + \sum_{i} \sum_{j} \pi_{ij}^s(\alpha + \xi)$$

$$\mathrm{FCQ}_i^r = \sum_{i} \rho_i^s(Q_i)$$

$$\mathrm{FCp}_i^r = \sum_{i} \sum_{j} \pi_{ij}^s(-\beta p_i)$$

$$\mathrm{FCe}_{ij}^r = \sum_{i} \sum_{j} \pi_{ij}^s(\gamma e_{ij})$$

During the process of adding optimality cuts, the lower bound θ will be decreased. The optimal solution of two stages will be generated when second stage objective is greater than θ. The optimality cut constraint in master problem (17) is:

$$\theta \leq \sum_{i} \sum_{j} \sum_{s} P_s \,.\, \omega_{ij}^s(\xi + \alpha) + \sum_{j} \sum_{s} p_s . \delta_i^s \,.\, D_j^{max} + \sum_{i} \sum_{j} \sum_{s} P_s \,.\, \gamma . \omega_{ij}^s . e_{ij} - \sum_{i} \sum_{j} \sum_{s} P_s \,.\, \beta \,.\, \omega_{ij}^s . p + \sum_{i} \sum_{s} P_s . \tau_i^s . Q_i$$

And then:

$$\mathrm{ORH}^v = \sum_{i} \sum_{j} \sum_{s} P_s \,.\, \omega_{ij}^s(\xi + \alpha) + \sum_{j} \sum_{s} P_s . \delta_j^s \, D_j^{max}$$

$$\mathrm{OCp}_i^v = \sum_{i} \sum_{j} \sum_{s} -P_s \,.\, \beta \,.\, \omega_{ij}^s . p_i$$

$$\mathrm{OCe}_{ij}^v = \sum_{i} \sum_{j} \sum_{s} P_s \,.\, \gamma . \omega_{ij}^s . e_{ij}$$

$$\mathrm{OCQ}_i^v = \sum_{i} \sum_{s} P_s . \tau_i^s . Q_i$$

The pseudo code of the proposed L-shaped for solving the proposed two stage stochastic programming model is represented in figure 3.

```
Set algorithm parameter: Max iteration, r (the number of feasibility cut), v (the number of optimality cut), and
θ (the upper bound of the primal sub-problem); r=0, v=0 and θ=+∞ ;

    FOR t=1:Max iteration
        Solve the master problem with objective function (16) subject to constraints (6), (7), (10)-(12),
        and (17)-(19) and obtain (X_i(t),Q_i(t), p_i(t), e_ij(t))

            FOR s=1 To scenario size
                Solve the primal sub-problem with objective function (20) subject to
                constraint (21)-(24) by considering X̂_i(t), Q̂_i(t), p̂_i(t), ê_ij(t);
                Solve the Feasibility check problem (FCP) with objective function (25)
                subject to constraint (26)-(29) by considering X̂_i(t), Q̂_i(t), p̂_i(t), ê_ij(t);
                    IF the dual objective of the FCP ≠ 0
                        Determine the feasibility cut (r) according constraint (18);
                        Add feasibility cut (r) to the master problem;
                        Set r=r+1;
                        Break FOR;
                    ELSE
                        feasibility cut (r)=0;
                    END
            END
        IF the second stage objective > θ and feasibility cut (r)=0;
            Obtain the optimality cut (v) according to constraint (17)
            Add optimality cut (v) to the master problem
            Set v= v+1;
        ELSE
            optimality cut (r)=0;
        END
            IF feasibility cut (r)=0 & optimality cut (r)=0;
                Break FOR
            END
    END
```

Fig 3. Pseudo code of the proposed L-shaped for solving the proposed model

5-Computational experiments and managerial insights

In order to validate the proposed model and the proposed solution methodology, some numerical examples are randomly produced. The parameter's values are presented in table 2.

Table 2. Computational study parameters

Parameter	Value	Parameter	Value
T	20000	dis_{ij}	~Uniform (10,100)
C	40	K	5
V	100	Cap_i	~Uniform (200,500)
μ	~Uniform (0.2,1.8)		

As mentioned in section 4, since the proposed model has a non-linear feature, commercial solvers of GAMS which are appropriate to solve the non-linear models should be used to solve the proposed model. Although, in large scale models, most of these solvers cannot guarantee the achievement of a global solution. Thus, in order to evaluate the efficiency of the L-shaped method the proposed method is compared with a commercial solver (BARON) in terms of government and facility location objective functions under different reward-penalty amounts. The results are displayed in table 3. All of the instances are solved applying commercial solver GAMS 23.5 on a core (TM) i5 computer with 2.40 GHz CPU and 4.00 GB RAM. In addition, to represent the demand function uncertainty, finite scenarios of the random component of demand function are randomly generated from Uniform (10, 1000).

Table 3. L-shaped algorithm and commercial solver results for the proposed MCLP model

(*I*,*J*,*N*,*S*)	**Instance**	**L-shaped algorithm**		**Commercial solver (BORON)**		**Commercial solver statues**
		Profit	**CPU time**	**Profit**	**CPU time**	
(10,10,3,10)	1	119727.27	0:00:33	119727.27	0:00:06	-
	2	121950.2	0:00:26	120966.7	0:00:09	Local solution
	3	125883.33	0:02:16	125883.33	0:01:12	-
	4	131383.33	0:02:23	122983.33	0:01:40	Local solution
	5	106866.67	0:02:14	104666.67	0:01:07	Local solution
(20,20,5,20)	1	1325150.33	0:05:51	1325150.33	0:17:07	-
	2	1698233.33	0:05:25	1238033.33	0:16:52	Local solution
	3	1964136.67	0:05:46	1964136.67	0:16:54	-
	4	2072906.06	0:05:19	1618571.52	0:16:53	Local solution
	5	2153448.33	0:05:54	1721355.33	0:17:21	Local solution
(50,50,10,50)	1	2896108.33	0:27:24	2266347.33	1:22:09	Local solution
	2	2665194.67	0:25:41	2317763.67	1:17:16	Local solution
	3	3041153.33	0:33:37	2965811.33	1:26:32	Local solution
	4	3334962.21	0:33:26	3036717.33	1:25:22	Local solution
	5	3187856.33	0:28:13	2818783.4	1:21:43	Local solution
(70,70,15,70)	1	5116931.33	0:54:17	4981744.33	2:13:27	Local solution
	2	5734855.12	0:46:54	5697701.12	2:06:15	Local solution
	3	6471032.33	0:51:18	6179672.33	2:17:42	Local solution
	4	6724755.06	1:11:07	6308833.33	2:21:24	Local solution
	5	6806334.33	0:56:42	6564813.27	2:17:39	Local solution
(100,100,30,100)	1	9772685.33	1:32:06	9667925.33	3:06:18	Local solution
	2	10615256.33	1:34:51	10184471.12	3:26:30	Local solution
	3	14088531.27	1:27:46	12635993.67	3:17:42	Local solution
	4	14768077.33	1:40:13	12829776.33	3:36:07	Local solution
	5	16470489.67	1:40:31	14379043.67	3:28:37	Local solution
Average		**4860556.34**	**0:37:36**	**4451874.88**	**1:27:46**	-

According to the obtained results in table 3, it is observed that the proposed L-shaped method has acceptable performance comparing to the commercial solver in MINLP proposed model in terms of solution quality and the CPU time as shown in figure 4. This Figure shows that in more than 80% instances, commercial solver obtains local solution with less objective function.

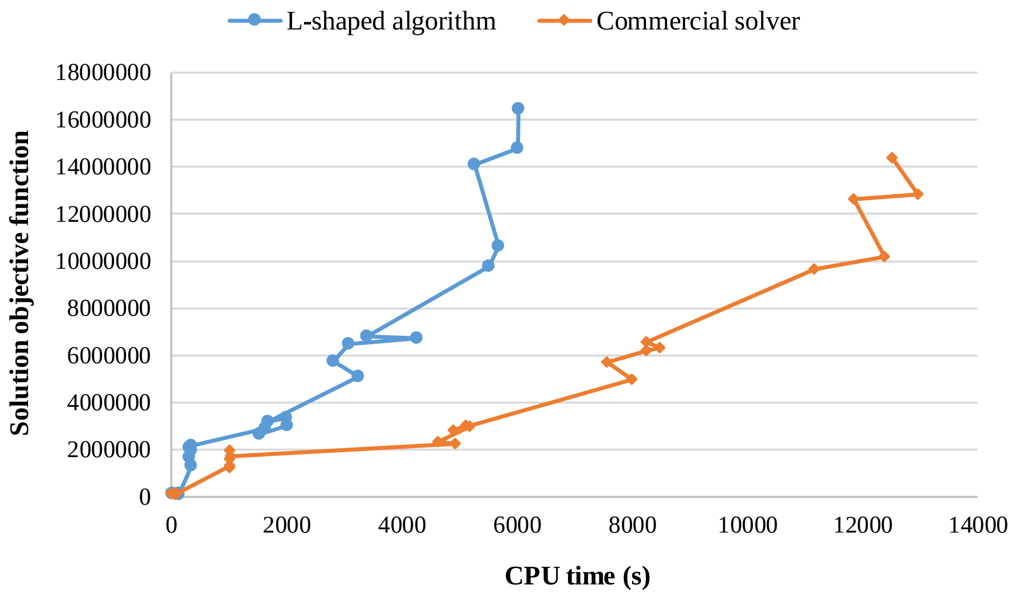

Fig 4. Comparing solution methodologies based on solution quality and CPU time

To evaluate the performance of the both proposed models, supportive model and legislative model, sensitivity on significant parameters are examined. Analytical sensitivity of these parameters will lead to the worth managerial results for both government and MCLP designer.

The reward-penalty mechanism is applied in the first model as a supportive tool of the government. This model is analyzed in terms of government desired target (T) and reward-penalty amount (λ). Firm's profit, customer welfare and government satisfaction indexes are explored influenced by changes in (λ, T). Government satisfaction and customer welfare measures are calculated with equation (30) and (31) formulas respectively:

$$Government\ satisfaction\ index = \sum_{\xi,i,j} p_\xi * \frac{Y_{ij}^{\xi}}{T} \tag{30}$$

$$Customer\ welfare\ index = \sum_{\xi,i,j} p_\xi * \frac{Y_{ij}^{\xi}}{(\alpha - \beta p_i + \gamma e_{ij} +\ \xi)} \tag{31}$$

In some cases, governments should intervene to lead firm decisions in supportive or legislative role to force or motivate them to ensure a minimum social welfare level index as well as a minimum profit of the firms. Thus, in this paper, these two indexes are prepared to explore the government's targets. First, the "customer welfare index" which calculates the expected production to the real demand accrued and second the "government satisfaction index" which calculates the expected production to the predefined government target. It is obvious that customers prefer that production be as near as to the real demand, but government based on customer welfare, firm limitations and firm profitability sets a predefined production target (T).

The selected instances for analyzing are listed in Table 3. The desired target of the instance (3) i.e. 20000 and reward amount of instance (2) i.e. 400 are considered as a base-case of the analysis. Actually, different instances with 10% increase and decrease in target and reward-penalty amount parameters have been generated. Table 4, shows the model sensitivity in terms of firm profit on government target and reward-penalty amount changes. It shows the model validity as well. Government satisfaction index represents the deviation of the service level with government target; positive deviation results gain profit and negative deviation loss for the firm. Thus, the firm's profit will be decreased by increasing government desired target for each amount of λ, due to fine for not reaching to the government desired target. It means that by growing government target and also decreasing the government satisfaction index, the firm's profit will be decreased. Besides, increasing in the reward-penalty amount leads to the higher profit when government satisfaction index is greater than 1 and otherwise leads to less profit. Considering instance (2) for each λ amount, since government satisfaction index is greater than 1, the firm will gain profit by containing reward. Thus, the firm's profit will be increased by increasing reward amount (λ). Otherwise, if government satisfaction index is less than 1 such as instance (3), the firm's profit will be decreased by growing the penalty amount (λ). As shown in Figure 5, firm profit with λ=440 gets the highest amount when government index is higher than 1 and also gets the least amount in government index (0.91, 0.83 and 0.76). In this analysis government satisfaction index, 0.91 is the breakpoint.

Table 4. Sensitivity analysis of (λ, T) in supportive reward-penalty model

λ	Instances	T	Government satisfaction index	Customer welfare index	Firm's profit
	1	16200	1.12	0.39	2741201.6
	2	18000	1.03	0.38	2194621
λ=360	3	20000	0.91	0.37	1474621
	4	22000	0.83	0.38	754621
	5	24200	0.76	0.37	-37379
	1	16200	1.12	0.4	2931021
	2	18000	1.02	0.38	2211021
λ=400	3	20000	0.91	0.37	1411021
	4	22000	0.83	0.38	611021
	5	24200	0.76	0.37	-268979
	1	16200	1.13	0.4	3019421
	2	18000	1.02	0.39	2227421
λ=440	3	20000	0.93	0.4	1258802
	4	22000	0.83	0.37	467421
	5	24200	0.76	0.36	-500579

It is worth to be mentioned that if MCLP model without government supportive role intervention considered, firm profit will be 1316200 and customer welfare index will be 0.36.

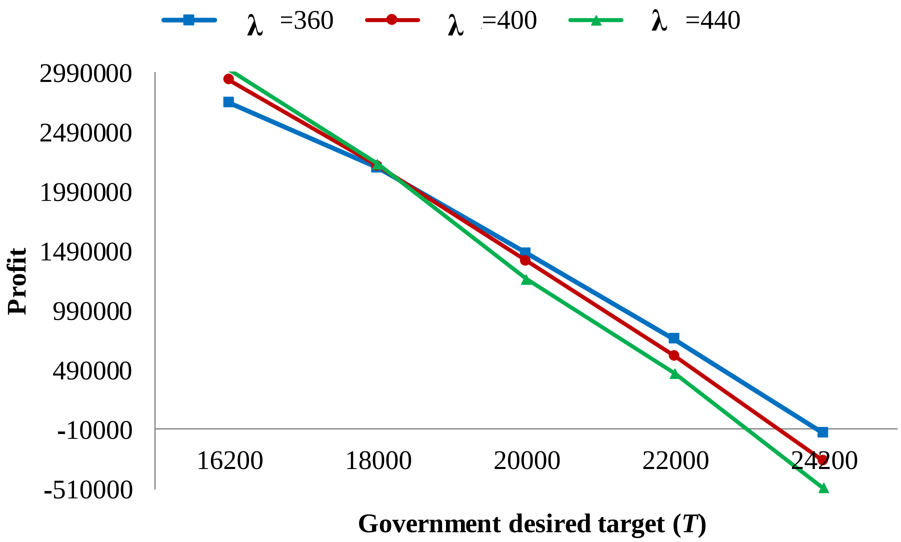

Fig 5. Supportive reward-penalty model sensitivity analyses in terms of (λ, T)

Generally it can be concluded by growing government target, the firm's profit will be decreased. Note that, until the government satisfaction index is greater than 1, it means that the firm's production is more that the desired level. Thus, λ is considered as reward in the proposed model. As shown in figure 5, until breakpoint for the lower λ, firm's profit is lower. On the other hand, when the government satisfaction index becomes lower than 1, firm's profit reduction rate is higher for bigger λ due to considering λ as penalty in the model because of losing demand.

In this study, the legislative model counterpart of the proposed supportive reward-penalty model is proposed in constraints (5)-(15). In this model, the government set a predefined satisfied demand level (L) for retailers as a constraint against a reward penalty model, which represents a compulsion for the MCLP designer. In this study predefined satisfied demand level is considered as equation (31):

$$L = \frac{T}{\sum_j D_j^{max}} \tag{31}$$

Since the objective of the MCLP is to locate a fixed number of facilities on a network of nodes and arcs to maximize the serviced demand nodes, N set as a parameter in the proposed model based on firm considerations. Thus in the legislative model, infeasibility of the model due to the high satisfied demand level is common (for N=3 and $L \geq 0.35$ model will be infeasible). Thus, in this sub-section sensitivity of the legislative model in terms of satisfied demand level and the number of facilities (L, N) is examined. The results are presented in Table 5. As shown in this Table 5, if the government is tended to reach higher satisfied demand level without any subsidies, he should facilitate firm to establish more facility center.

Table 5. Sensitivity analysis of (L,N) in legislative model

N	Instances	T	L	Firm's profit
N=4	1	16200	0.37	2425848.9
	2	18000	0.41	1555650.4
	3	20000	0.45	Infeasible
	4	22000	0.6	Infeasible
	5	24200	0.7	Infeasible
N=5	1	16200	0.37	4402632
	2	18000	0.41	4016177
	3	20000	0.45	3848804.8
	4	22000	0.6	Infeasible
	5	24200	0.7	Infeasible
N=6	1	16200	0.37	4898803.6
	2	18000	0.41	4898803.6
	3	20000	0.45	4729730
	4	22000	0.6	2219365.6
	5	24200	0.7	Infeasible

According to the aforementioned sensitivity analysis, some managerial insights can be obtained which are mentioned as follows:

- If the government implements incentive mechanisms such as reward-penalty, subsidy or tax discount then the MCLP designer will be tended to increase customer welfare index and service level.
- Government should carefully consider all the implications of enacting rules and regulations to ensure that they are appropriate for the circumstances and provide benefits enough to the consumers and companies, simultaneously. So the government can reach to this purpose by reducing the MCLP costs in the supportive model or facilitating using more facility centers for the MCLP in legislative model counterpart.

6-The expected value of perfect information and the expected value model

In order to deal with the uncertainty of the demand function in the proposed model, expected value (EV) approach is applied as one of the first simple approaches in solving this type of problems. In fact, in this approach, each uncertain parameter is replaced by the expected value of the parameter in different scenarios. Since this approach applies only an average of each uncertain parameter, infeasibility occurrence will be so probable. Hence, to evaluate the performance of the EV solution (EEV), the result of the first stage variables of the EEV was fixed in the EV model and then the model feasibility was evaluated for each scenario separately (it is reported as feasibility probability in table 6). Value of stochastic solution index (VSS) is used to evaluate the difference between stochastic programming solution and EEV. Besides, to determine the amount of reasonable investment on demand function prediction, perfect information (PI) model value which called wait-and-see (Madansky 1960) value should be obtained. The Wait-and-see value is the expected of all these optimal values which can evaluate the expected performance of complete information. In wait-and-see approach prior information of the demand function is known, thus, decisions will be taken simultaneously. The expected value of perfect information (EVPI) is used to evaluate the difference between stochastic programming solution and perfect information solution. Thus, this is an upper limit of the reasonable payment in return for complete information about the future. The results to test the performance of the stochastic programming solution comparing EEV and PI solution are reported in table 6.

Table 6. Comparing the Expected value, stochastic programming and prefect information approach's results

Instance	EEV		Stochastic programming (SP) profit	Perfect information (PI) profit	VSS = (EEV-SP)	EVPI = (PI-SP)
	Profit	Feasibility probability				
Instance 1	119016.6	80%	119727.3	121706.3	189.3	1979
Instance 2	124394	100%	121950.2	126447	2443.8	4496.8
Instance 3	126450	60%	125883.3	130155.3	566.7	4272
Instance 4	133781	60%	131383.3	136422.6	2397.7	5039.3
Instance 5	107666.7	80%	106866.6	108976.7	800.1	2109.4

When we bring the EEV to each random component of demand function scenario, we find that the EEV couldn't meet all of the demand scenarios. As it is reported in Table 6, in most of the instances, the constraints will be violated. It also means there is no feasible EEV, as it is shown in instances 1, 3, 4 and 5. However, in our work, the stochastic programming solution can ensure the rescue effect by hard constraints. Besides, Stochastic programming approach leads the firm to take a more conservative decision to decrease the infeasibility risk.

6-Conclusion

In this study, the effects of governmental policies in a supportive role and legislative role on a maximal covering location problem facing stochastic demand were examined. Government considered as a legislative and authorized entity which can lead companies to produce enough product and ensure responding the desired amount of the demand. For this purpose, a reward-penalty two-stage stochastic programming model was proposed to cope with an uncertain component of the demand function. Demand function was considered to be influenced by the retail price and facility sales effort levels as a pricing and advertising linear demand function to sets retail prices for each opened facilities and various level efforts based on zone's attractiveness.

The proposed model is more suitable for essential products such as food and medicine with low price because of legal restrictions to increase; while companies are not interested in producing products and governments should intervene due to the social issues to force or motivate them to produce the desired level of the products. Indeed, we suppose that increasing the social welfare level is the aim of the government, and government does not take any financial advantages.

To cope with non-linearity term of the proposed MINLP model, the L-shaped algorithm was applied. Numerical examples were randomly generated and used to evaluate the solution method efficiency. Computational results showed that the L-shaped method has acceptable performance comparing to the commercial solvers in terms of solution quality and the CPU time. Besides, the sensitivity of the both supportive and legislative proposed models to the critical parameters was examined to illustrate the governmental policies impacts the MCLP model. As results, some managerial insights due to the government policies effect on customer welfare and firm benefit were proposed. Government should carefully set all the enacting rules and regulations to ensure that they are appropriate for the circumstances and provide benefits enough to the consumers and firms, simultaneously.

Moreover, the EV model was given and corresponding EV solution was compared with the stochastic programming solution in terms of the firm's profit and feasibility probability. Furthermore, the EVPI was calculated to assess the value and necessity of obtaining perfect information of stochastic demand function.

Also considering more factors such as the price of substitutes or complements goods and services, income levels, time of delivery, bundling and tying, social conformity and nonconformity, environmental concerns, in the demand function and applying the proposed model for a real case study can be employed as a future sight. Using real case study can lead the authors to achieve more reliable predicted demand function based on previous data. Also using both stochastic and robust approaches in MCLP and government decisions in a leader-follower configuration can be considered as another direction for the future works.

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